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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

INTEGRATED SYSTEMS OF AUTOMATIC RADIO EQUIPMENT

by

M.P. Bobnev, B.Kh. Krivitskiy, M.S. Yarlykov

FOREIGN TECHNOLOGY DIVISION

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INTEGRATED SYSTEMS OF AUTOMATIC RADIO EQUIPMENT

By: M.P. Bobnev, B.Kh. Krivitskiy, M.S. Yarlykov

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

Integrated systems of automatic radio equipment.

M. P. Bobnev, B. Kh. Krivitskiy, M. S. Yarlykov.

Page 2.

In the book are examined questions of the aggregation of the radio engineering servo systems with the non-radiotechnical meters. Are given the characteristics of the most commonly used non-radio engineering meters, are examined the special features/peculiarities of the unification of separate meters into the single system, are revealed/detected the physical causes for gain in accuracy and freedom from interference of integrated systems and they are given some quantitative evaluations/estimates. Are described integrated navigation aids, meters of angular coordinates, range and speed. Are given some results of the experimental investigations, which make it possible to rate/estimate the real possibilities which give integrated systems in comparison with the simples.

The book is intended for wireless engineers, who are interested in questions of the construction of radio engineering coordinators, and also for the engineers on the automation, who work in the area of exploration of the radio engineering servo systems.

Is assumed the acquaintance of the theory of automatic control by readers with the bases in the space of the course of technical VTUZ [higher technical educational institution].

Page 3.

Preface.

In this book are examined questions of the construction of the automatic meters, united for the purpose of an increase of the accuracy of the determination of the target coordinates into the single integrated system, moreover one of the meters is radio engineering servo system, and by others - meter of the parameters of the proper motion of object. Similar systems find use for determining the coordinates of the moving targets from onboard of flight vehicle (or another mobile object)or for purposes of navigation.

The book is based on those published in the literature materials, which are processed and given into the system.

The authors express their gratitude to the reviewers of the book to G. I. Gorgonov, to Yu. P. Borisov and to S. V. Pervachev.

The small size of the book forced the authors to present some questions in abbreviated form, and sometimes also it is concise.

In connection with the fact that the book on a similar theme is

published for the first time, it, apparently, is not deprived of deficiencies/lacks, including of methodological character.

The authors will be grateful to the readers who will consider it possible to express wishes on an improvement in the book.

Page 4.

Introduction.

For the solution of the problems of navigation and control of the mobile objects (aircraft, rockets and ships) is utilized the information about coordinates and parameters of motion, obtained with the aid of the meters of different types. Meters differ in the utilized physical laws, to the coordinate system, in which are determined one or another navigational parameter, accuracy, to the statistical characteristics of errors, etc. If all meters work independently, i.e., are not united into the navigational complex, the aircrew or ship for the solution of the problem position findings or speed of Budde of using predominantly one of the meters, for example, which ensures the greatest accuracy. Readings/indications of other meters are utilized only for the confirmation of the reliability of the measured coordinates or for eliminating the ambiguity of reading. Thus, for instance, information about flight altitude can be obtained with the aid of the barometric and radio altimeters, the data about the flight speed - with the aid of the sensor of airspeed and Doppler meter of ground speed, etc.

The presence of several instruments does not aboard yet mean

that the system of navigation (or control) is complex. Complex are the systems where in the process of definition of one coordinate are utilized signals not less than two meters.

One of the earliest integrated systems is gyromagnetic compass. In this instrument are united two independent meters, action of which is based on different physical phenomena and errors of which have sharply different statistical characteristics.

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One of these meters (sensor of the magnetic heading) determines with the high accuracy the average/mean value of the position of the axis of aircraft relative to the magnetic meridian. However, the signal of this sensor possesses large and comparatively broadband - fluctuation error. Another meter - gyroscopic sensor - on the contrary, possesses the very narrow spectrum of errors, but the average/mean value of its signal changes in the time due to the precession.

The unification of signals of both sensors into the single system - gyromagnetic compass - makes it possible to raise the general/common/total accuracy of the measurement of the course of aircraft.

Aggregation finds wide application, also, when one of the meters is radio engineering. As an example can serve the stabilization of the antenna systems of the radio engineering self-homing heads of rockets with the aid of the signals of gyroscopic sensors. Here is realized the aggregation of the meters, which work on different principles. Gyroscopes measure the position of longitudinal axis in the inertial space, and radio engineering direction finder determines target position relative to this axis. This separation of functions proves to be highly useful. Complex angle measuring devices/equipment solve the problem standing before them considerably more successfully than the devices/equipment, which work independently.

Complex can be whole measuring systems, for example navigational.

The fundamental problem, which solve the integrated systems of navigation and control, consists of the joint use for purposes of an increase in the accuracy of the measurement of the information, obtained with the aid of different ones, that are on board the meters.

Radio engineering devices/equipment possess the essential deficiency/lack: their work can be disrupted by radio interferences. With an increase in the velocities of flight vehicles grow the

requirements for the dynamic accuracy of radio engineering devices/equipment, which contradicts the requirement of an increase in the freedom from interference, since it requires the expansion of the passbands of servo systems.

With the aid of the radio engineering servo system is realized the measurement of the parameters of the mutual motion of mobile object and target. The parameters of proper motion are measured with the aid of the devices/equipment, available on board this object.

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This information, obtained by meter of the parameters of proper motion (ISD) is a priori for the system, which is determining the parameters of mutual motion. By using the a priori information of ISD to a considerable degree to solve contradiction between the required high dynamic accuracy of radio engineering system and its filtering properties, i.e., it is possible to throttle/taper passband without a decrease in the dynamic accuracy.

The meters of the parameters of proper motion (ISD) work on different principles. Most frequently as ISD are utilized inertial navigation systems, gyroscopic meters of the angular oscillations of mobile objects, aerodynamic meters of airspeed, Doppler meters of

ground speed and drift angle, etc. The necessary condition of gain in the accuracy during the mutual unification (aggregation) of meters is difference in the spectral characteristics of the errors of these meters this difference exactly there is at the aggregation of the radio engineering meters of the parameters of its own and mutual motion with accelerometric and gyroscopic meters of the parameters of proper motion.

The problem of the rational selection of the composition of meters and their optimum unification into the single complex, which has the specific designation/purpose, is very complicated and it is at the present time distant from the final solution. Is at the same time now accumulated certain experience of the creation of integrated systems. On the base of the analysis of the utilized systems and the results obtained in the process of their tests has the capability to formulate some general/common/total principles of the solution of the problem of aggregation and to rate/estimate the advantages of integrated systems.

On the content and method of the presentation of the published material the decisive effect showed/rendered that the fact that this book is the first attempt at generalization and systematization of sometimes contradictory materials on the aggregation. The authors attempted to introduce the readers consecutively/serially into the

circle of the ideas, connected with the aggregation, to show the possible methods of solution of problem, and to come to light/detect/expose the advantages which gives aggregation.

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The small size of the book did not make it possible to develop separate positions in more detail.

Thus, for instance, barely are touched upon questions of the translation of errors during the use in the integrated system of the signals, obtained in different coordinate systems, questions of the effect of the nonlinearity of meters themselves and devices/equipment of the translation of coordinates, etc. Are omitted questions of the experimental analysis of integrated systems, although some quantitative results of this analysis were reflected in the book.

Taking into account that the questions touched upon in the book interest the wide circles of the specialists of both radio engineering and other specialties, the authors attempted to utilize the mathematical apparatus, well known to wireless engineers and to specialists in automation, general considerations to illustrate by specific examples. It was at the same time considered that the accuracy analysis and freedom from interference even the simplest from the point of view idea and the circuit execution of radio engineering system with the correction is fairly complicated problem.

Therefore it is not possible to be restricted to the framework of the simple linear theory of systems with the constant parameters. For the illustration of space and content of the analysis of complex radio engineering meters as systems with the random parameters, and also the demonstration of real gain in the freedom from interference and the accuracy, caused by aggregation, in the book is given an example of the analysis of the pulse automatic range finder, corrected by the signals of the sensor of airspeed.

The authors will be grateful to the readers who will make their observations about the questions, touched upon in the book.

Page 8. (No typing).

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Chapter 1.

FUNDAMENTAL CHARACTERISTICS OF THE METERS OF INTEGRATED SYSTEMS.

The fundamental meters of the parameters of the mutual motion of flight vehicle and target are the automatic radio engineering meters, with the aid of which they are determined:

- the angular coordinates of targets;

- distance between the flight vehicle and the target;

- mutual velocity of the displacement/movement of target and apparatus.

For measuring the parameters of proper motion of flight vehicle can be utilized diverse navigational devices, for example, the Doppler meter of ground speed and drift angle (DISS), radio compass, the inertial system of dead reckoning, inertial-celestial navigation

system, etc.

All these devices/equipment can be divided into the radio engineering ones and the non-radiotechnical ones. The action of non-radiotechnical meters is based on the use of primary meters (sensors). The greatest use/application they at present found:

- sensors (meters) of air speed of flight vehicle;

- gyroscopic sensors;

- accelerometric sensors.

In this chapter are briefly examined some characteristics of meters, which have vital importance for the analysis of integrated systems.

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§ 1.1. Automatic radio engineering meters.

Radio engineering meters are the servo systems whose action is based on the use of radio waves, which arrive from the tracked a target. These meters in many respects determine properties and

characteristics of the integrated systems in which they are included. Radio engineering meters (or the radio engineering servo systems) are utilized for measuring the parameters of mutual and proper motion of flight vehicles. They serve for determining the angular coordinates of different objects (automatic tracking in the direction - ASN), ranging (automatic range control - ASD), and also for the automatic determination of the relative velocity of objects. In latter/last meters is included the automatic servo filter whose action is based on the automatic frequency control (APCh) of the servo self-excited oscillator (frequency or phase).

Any radio engineering servo system is located under the effect of the external agencies which are divided into those controlling and exciting (interference). Controlling actions should be reproduced to most accurately by system; the mixing effects, on the contrary, be suppressed to most fully by this system. Two types of the mixing effects are general/common/total for all radio systems. These are - the radio interferences, which enter together with the signal the input of radio receiving equipment and the internally-produced noise of this radio receiver.

Assuming/setting the operating principle of fundamental radio engineering meters known [see 12], let us examine only the characteristic features of each system, and then let us pause at the properties, which are general/common/total for all radio engineering systems and which differ them from the automatic systems of other types.

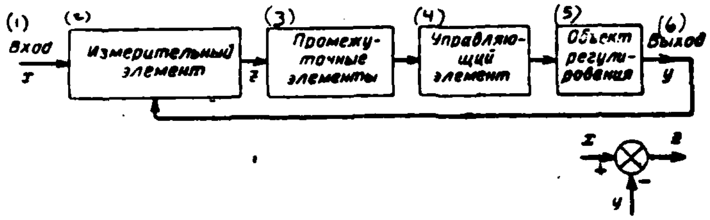

Fig. 1.1.

Key: (1). Input. (2). Measuring element/cell. (3). Intermediate means. (4). Control device. (5). Controlled system. (6). Output.

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The functional diagram of the radio engineering servo system can be brought to the general/common/total functional diagram in Fig. 1.1, where disagreement/mismatch z is formed as the difference of input x and output (adjustable) y of the values:

$$z=x-y.$$

However, the input of radio engineering system (antenna, receiver, etc.) enter not value x, radio signal (or high-frequency oscillations). This has important value and is the distinguishing feature of the radio engineering servo systems.

Briefly let us pause at the special features/peculiarities of

each of the systems enumerated above.

a) the system of automatic tracking in the direction.

The input of servo system is angle γ between the direction to the object from which are received the radio signals (for example, the homing station, ship, ground target), and as adjusting axis (for example, by the longitudinal axis of the aircraft where is established/installed system).

The output of system is angle γ_{H}, on which it is turned the equisignal direction ON relative to the same axis (Fig. 1.2a). This rotation is realized most frequently with the aid of actuating motor or gyroscopic device to which are fastened/strengthened the elements of antenna system.

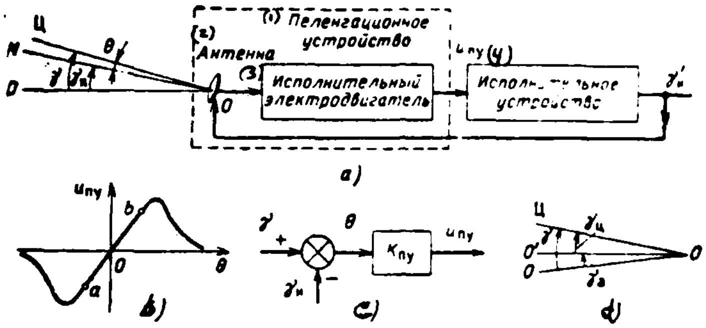

Fig. 1.2.

Key: (1). Direction-finding device/equipment. (2). Antenna. (3). Actuating electric motor. (4). Actuating element.

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Sometimes the rotation of equisignal direction is reached by changing the electrical state of the elements of antenna system. Signals, which affect the actuating element in which is included the servomotor, are formed as a result of the transformation of voltage/stress $u_{пу}$ of direction-finding device/equipment.

In the direction-finding device/equipment the entering the input antennas radio signals are converted into the voltage/stress, which depends on disagreement/mismatch $\theta=\gamma-\gamma_и$.

Dependence $u_{\text{пу}}(\theta)$ is the direction-finding characteristic (Fig. 1.2b) and takes the form of curve with the central symmetry. (For simplici ty there is not considered the effect of minor lobes)

Value

$$K_{\text{пу}} = \left(\frac{du_{\text{пу}}(\theta)}{d\theta}\right)_{\theta=0} \qquad (1.1)$$

is called the transmission factor of direction-finding device/equipment or steepness direction-finding characteristic.

For the linear section ab of characteristic transformation θ in $u_{\text{пу}}$ it is possible to represent by elementary block diagram (Fig. 1.2c), moreover:

$$u_{\text{пу}} = K_{\text{пу}}(\gamma - \gamma_{\text{н}}) = K_{\text{пу}}\theta. \qquad (1.2)$$

The inertness of direction-finding device/equipment, as a rule, can be disregarded/neglected; as a rule, it is possible to disregard; when account, for inertness is necessary, is conveniently this inertness related to the subsequent elements of system.

The control pressure in the system ASN is the angle γ, which consists of angular displacement of object (target) $\gamma_{\text{ц}}$ and angle of rotation $\gamma_{\text{в}}$, of the caused by motion flight vehicle (mobile object), on which is established/installed the system ASN (Fig. 1.2d):

$$\gamma = \gamma_{\text{ц}} + \gamma_{\text{в}}.$$

Angle γ_a can be determined with the aid of the meter, established/installed on board the mobile object (ISD) and then to introduce into the radio engineering system.

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Disturbances/perturbations in the systems ASN are:

- radio interferences, which enter together with the signal the input and the internally-produced noise of radio receiver;

- angular noise of target;

- amplitude noise of target;

- noises of the servo system, caused by the inadequacy of its elements/cells (stagnation of engine, gap, nonlinearity in the mechanical linkages, etc.).

- imbalance in the dc amplifiers and the phase discriminators (zero drift).

Let us examine briefly the characteristics of the enumerated disturbances/perturbations. Internal noise, just as the interference, which enters the input of radio receiver together with the radio signal, has the wide spectrum, which substantially exceeds the passband of closed system. Exception is the case of electronic jamming when their spectrum can have narrow band.

Confident lockon is realized, if the ratio of the amplitude of impulse/momentum/pulse to the rms voltage/stress of the internal noise in the band of radio receiving equipment does not fall below 3-4. Under these conditions the error, caused by internally-produced noise, does not have vital importance.

At the base of the onset of the amplitude [1] and angular noise of target lies/rests one and the same physical phenomenon.

FOOTNOTE [1]. Sometimes amplitude noise is called also amplitude fadings. ENDFOOTNOTE.

As a result of the summation of the oscillations, which arrive into the location of antenna system from different points of the reflecting object, forming complicated interference pattern. Field force and direction to "instantaneous center of reflection" (i.e. the point for which the error signal it is equal to zero, if to it is

directed equisignal line) do not remain constant/invariable, but chaotically they are changed in view of many reasons (for example, the oscillations of objects, instability of the carrier frequency).

The changes in the field force, caused by these reasons, are called amplitude noise, the angular displacements of the effective center of reflection - by angular noise of target. Amplitude noise has an effect on systems with the consecutive comparison of signals and the amplitude direction finding (system with the conical scanning); to the effect of angular noise are subjected the systems of all types.

The correlation function of the modulation factor $m_{\text{п}}$ of amplitude noise according to the data [18] and [14] for the propeller-driven aircraft can be described by the expression

$$R_{m\text{п}} = \sigma^2_{m\text{п}} e^{-\alpha_m|\tau|} \cos \Omega_m \tau, \qquad (1.3)$$

where $\sigma^2_{m\text{п}}$ =0.2-0.3 - dispersion of modulation factor, caused by amplitude noise; α_m; Ω_m - values, which depend on the property of target and which are of the order α_m =20-25 cek^{-1}(neq) $\Omega_{m\text{п}}$ =40 cek^{-1}.

Later investigations [27] make it possible to make a conclusion about the admissibility of the approximation of correlation function (for the jet aircraft) the more simple expression

$$R_{m\text{п}}(\tau) = \sigma^2_{m\text{п}} e^{-\alpha_m|\tau|}. \qquad (1.4)$$

The corresponding spectral densities of the modulation factor of amplitude noise are expressed by relationships/ratios

$$S_{m\text{п}}(\omega)=\sigma^2_{m\text{п}}\alpha_m\left[\frac{1}{(\Omega_m-\omega)^2+\alpha^2_m}+\frac{1}{(\Omega_m-\omega)^2+\alpha^2_m}\right], \quad (1.5)$$

for the correlation function of type (1.3) and

$$S_{m\text{п}}(\omega)=\frac{2\alpha_m\sigma^2_{m\text{п}}}{\alpha^2_m+\omega^2} \quad (1.6)$$

for the correlation function of type (1.4). The effective one-way bandwidth in the latter case can be rated/estimated by value

$$B_m=\frac{1}{G_{m\text{п}}(0)}\frac{1}{2\pi}\int_0^\infty G_{m\text{п}}(\omega)\,d\omega=\frac{\alpha_m}{8}, \quad (1.7)$$

where

$$G_{m\text{п}}(\omega)=2S_{m\text{п}}(\omega).$$

When $\alpha_m=20\ сек^{-1}$ we obtain B_m=2.5 Hz.

According to available data [14, page 224] the spectrum of amplitude fluctuations is wider than the obtained value (on the order of 6-10 Hz).

The action of amplitude noise in the systems ASN with the conical scanning is developed in the appearance of supplementary error angle $\Delta\theta_\text{п}$, of that added to fundamental disagreement/mismatch $\theta=\gamma-\gamma_\text{и}$ (1.3a).

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Spectral density $S_{\theta\text{п}}$ of angle $\Delta\theta_\text{п}$ can be expressed through the spectral density of the modulation factor of interference by the

following formula [12, page 299]:

$$S_{\theta n}(\omega)=\frac{1}{K_m^2}[S_{mn}(\omega-\Omega)+S_{mn}(\omega+\Omega)], \qquad (1.8)$$

where K_m - conversion factor of antenna installation, equal to the slope/transconductance of radiation pattern in the equisignal direction [12, page 219]; Ω - frequency of scanning diagram.

The angular noise of target produces the errors in the systems ASN of any type and it is developed in the form of complementary angle γ_{ym}, of that added to the inlet angle γ, so that input signal upon consideration of angular noise will be angle $\gamma+\gamma_{ym}$ (Fig. 1.3b).

The statistical properties of angle γ_{ym} can be approximately described by the correlation function

$$R_{ym}(\tau)=\sigma_{ym}^2 e^{-\alpha_{\gamma m}|\tau|} \qquad (1.9)$$

and the spectral density

$$S_{ym}(\omega)=\frac{2\alpha_{\gamma m}\sigma_{ym}^2}{\alpha_{ym}^2+\omega^2}, \qquad (1.10)$$

corresponding to it where α_{ym} - value, which depends on the properties of target and which has the same order as α_m; σ_{ym}^2 - dispersion of angular noise.

According to [14, page 479, 488] it it is possible to approximately design from the formula

$$\sigma_{ym}=0{,}15\frac{L}{Д}(рад), \qquad (1.11)$$

where L - significant dimension of target (for example, the length of

ship); D - target range.

By noises of servo system are understood the random errors, caused by the inadequacy of separate elements/cells (gaps, nonlinearity, stagnations, etc.).

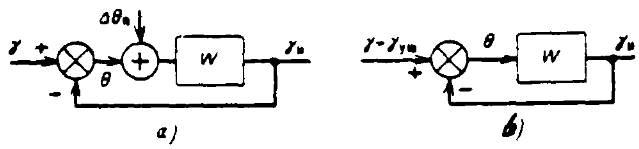

Fig. 1.3.

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The value of these errors depends on technology of production and structural/design execution, mainly, the electrical nodes of system and can be brought to the tolerance level.

Unbalance in the dc amplifiers and the phase discriminators of system leads to the appearance of slowly changing following error; its value is less, the greater the transmission factor of the components, which precede the amplifiers in which appears the drift.

Usually system ASN has astaticism of the 1st order, i.e., contains in the given duct/contour one integrating component/link, although they can utilize systems with astaticism of the 2nd order. With an increase in the order of astaticism become complicated the problems of the stabilization of system and guarantee of the necessary dynamic characteristics.

b) The system of automatic range control.

The widest application find pulse range-only radar, in which is realized automatic tracking the impulse/momentum/pulse, reflected from the target. The functional diagram of this system (Fig. 1.4a) consists of temporary/time discriminator, intermediate means (PE) and device/equipment of time delay with the transmission factor K_y.

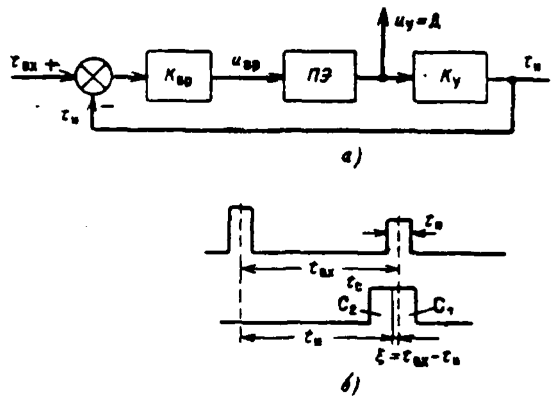

Fig. 1.4.

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In the system is created the pair of the selector (servo) impulses/momenta/pulses C_1 and C_2 time delay t_u of which relative to sounding pulse is changed proportional to control voltage u_y

$$t_u = K_y u_y. \qquad (1.12)$$

The intermediate means of system (integrators, amplifiers, filters) are intended for imparting to system the necessary dynamic properties.

Pulse range-only radar relate to the class of discrete/digital automatic control systems. Pulse element/cell in the system is time discriminator (VR).

Into composition VR enters time discriminator where time disagreement/mismatch $\xi=t_{\text{вх}}-t_{\text{н}}$ (Fig. 1.4b) is converted into the surge voltage or the current. The latter in the smoothing or integrating element/cell VR is converted into the error signal. The majority of the in practice utilized systems ASD has such parameters, which competent replacement of discrete/digital system by certain equivalent continuous system see [12]. During this replacement of time discriminator of any type can be replaced with the converter of temporary/time disagreement/mismatch ξ into certain averaged during the repetition period $T_{\text{п}}$ voltage $u_{\text{вр}}$:

$$u_{\text{вр}}=\varphi(\xi)$$

and by the smoothing or integrating elements/cells.

The latter are convenient to relate to by the intermediate means of system.

Function $\varphi(\xi)$ (Fig. 1.5a) possesses central symmetry, with what value

$$K_{\text{вт}}=\left(\frac{d\varphi(\xi)}{d\xi}\right)_{\xi=0}\frac{в}{мксек} \qquad (1.13)$$

it is called transmission factor VR.

In the linear section ab of characteristic VR it is possible to

represent by the equivalent diagram in Fig. 1.5b, in which

$$u_{B1}=K_{B1}\xi=K_{B1}(t_{BX}-t_{H}). \qquad (1.14)$$

The operations/processes of smoothing (S) or integration (I), which are realized by one or the other concrete/specific/actual diagram VR, are related to the subsequent (intermediate) elements of system.

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System ASD, as a rule, has astaticism of the 2nd order, i.e., contains in the given equivalent diagram two integrators. The stabilization of this system does not present great difficulties, since in the system usually are absent the electromechanical elements/cells with the considerable inertness.

The use of systems with astaticism of the 2nd order has large common sense, since besides the signal of range here it is possible to obtain the signal of velocity.

By the control pressure in the system will be the input range

$$Д_{BX}=\frac{c\,t_{BX}}{2}. \qquad (1.15)$$

being the distance between the object and the target, or time t_{BX}.

Distance $Д_{BX}$ consists of two components:

$$Д_{вх}(t)=Д_{ц}(t)+Д_{а}(t) \quad (1.16)$$

or $t_{вх}=t_{ц}+t_{а}$, corresponding to the displacement/movement of target $Д_{ц}$ and object itself $Д_{а}$ relative to to fixed coordinate system (for example, the Earth).

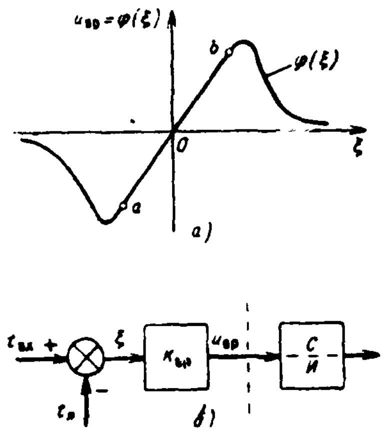

Fig. 1.5.

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As in the preceding case, this examination is of interest from the point of view of aggregation, since one of these components ($Д_a$ or t_a) can be determined with the aid of the meter of proper motion and then it is introduced into the radio system.

Disturbances/perturbations in the system ASD are:

- internally-produced noise of receiver and ambient noise on the radio frequency. The latter, as a rule, is wide-band for exception of the case of electronic jamming;

- instability of the device/equipment of time delay;

- fading of signals (amplitude noise of target);

- fluctuation displacement of the center of reflections, caused by the complicated character of target; usually these displacement does not fall outside the boundaries of target. For the group target these fluctuations can be very considerable.

Let us note that the system ASD frequently is utilized in order to accomplish the selection of target, i.e., to isolate necessary target against the surrounding background and to ensure the passage through the radio receiver of signals only from this target.

c) The system of the automatic frequency control.

From the diversity of the problems, performed by systems APCh, let us isolate two: the determination of Doppler frequency in the systems of the automatic determination of the vector of ground speed and relative rate of closure with the target (in the systems of selection on the velocity). The input of system APCh enters the signal in which useful information is included into the displacement

it is determined with the aid of the narrow-band servo system. Signal can be harmonic or narrow-band noise. In the latter case the tracking is realized after the central frequency of the noise spectrum, proportional to the velocity of object. This problem is performed by frequency or phase system APCh [33].

If in the Doppler meter is utilized frequency system APCh, then it is usually selected astatic. Into its composition enters the supplementary integrator which is switched on between the FM discriminator and the control of frequency.

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The important element of system APCh is the frequency discriminator (detector)whose characteristic takes the form of curve with the central symmetry (Fig. 16a). In section ab of characteristic output voltage/stress $u_{чд}$ is proportional to the divergence of frequency f of input from the transient frequency f_0 of discriminator (Fig. 1.6b):

$$u_{чд}=K_{чд}(f-f_0), \qquad (1.17)$$

moreover the coefficient

$$K_{чд}=\left(\frac{du_{чд}}{df}\right)_{\Delta f=0}, \qquad (1.18)$$

where $\Delta f=f-f_0$ it is called the transmission factor of frequency discriminator.

Characteristic (Fig. 16a) relates to the harmonic input signals. Qualitatively picture is not changed also with the narrow-band noise signals.

The control pressure is Doppler frequency. In this case total frequency switch $F_д$ is caused by two reasons: however, by the motion

of target (frequency switch $F_{\text{ц}}$) relative to certain coordinate system and by the displacement/movement of object relative to the same system:

$$F_{\text{д}} = F_{\text{ц}} + F_{\text{а}}. \qquad (1.19)$$

The speed of flight vehicle (and value $F_{\text{а}}$) proportional to it can be determined with the aid of the meter of proper motion and introduced as the supplementary signal into the servo filter.

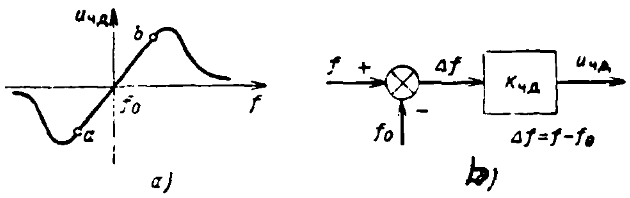

Fig. 1.6.

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The mixing effects in the system APCh are the interferences, which go together with the signal, the instability of the frequency of internal generators, and also fluctuation of input signal. The system of phase APCh which is applied in the Doppler meters, it does not differ from usual. Its properties are widely illuminated in the literature, for example in [50, 51], and therefore here it additionally is not examined.

Let us pass to the examination of the general/common/total special features/peculiarities of the described systems.

The block diagrams (Fig. 1.2c 1.5b, 1.6b), which relate to the action of discriminators, only formally reflect the operation/process of the comparison of values γ and $\gamma_н$, $t_н$ and $t_н$, f and f_0 and the conversion of these values into the voltage/stress, proportional to

disagreement/mismatch.

This representation, although it is very useful, since makes it possible to use for the investigation of the properties of systems the rich arsenal of the means of the theory of automatic control, it does not reflect the essence of processes in the measuring elements/cells. These processes are incomparably more complicated than the simple operation/process of subtraction with the subsequent amplification, and the representation accepted about the discriminator needs refinement.

A question about the representation of real measuring devices by equivalent ones for the purpose of convenience in the analysis of systems was examined in [12] and [15]. In the majority of the practical cases to convenient and sufficient general/common/total ones is the equivalent, examined below.

Let us accept subsequently for the input and monitored values (angular displacement, temporary/time time lag, the frequency of oscillation/vibrations, etc.) the general designations x and y respectively, and disagreement/mismatch in the system let us designate through z=x-y.

For each value of disagreement/mismatch z output voltage/stress

is distinguished u_p under the effect on the input of the radio receiver of interference in the form of the broadband (white) additive noise (with spectral density $S_{\text{вх}}$) it is random function of time, which it is possible to represent in the form of the sum of average/mean value $\langle u_p \rangle$ and centered random function ξ(t). In comparison with the passband of the subsequent elements of set of functions ξ(t) has very wide spectrum and it it is possible to identify with δ - by correlated noise with the spectral density constant in the band of system S_p.

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Average/mean value $\langle u_p \rangle$ depends that disagreement/mismatch z, the jamming intensity at input $\Pi_{\text{вх}} = \sqrt{S_{\text{вх}}}$ and of signal amplitude $U_{\text{вх}}$ at the input of radio receiving equipment.

Thus,

$$u_n(t) = \langle u_p(z, \Pi_{\text{вх}}, U_{\text{вх}}) \rangle + A\xi(t). \qquad (1.20)$$

Here ξ(t) - white noise of the single spectral density

$$A = \sqrt{S_p}. \qquad (1.21)$$

Intensity of noise A is in the general case function z and it it is possible to present in the form of the series/row

$$A = A_0 + A_1 z + A_2 z^2 + \cdots \qquad (1.22)$$

In [1] it is noted, that the spectral density of equivalent noise is the even function z. Being limited to two terms of expansion (1.22) in the series/row, we will obtain

$$S_{ш} = S_0 + S_2 z^2. \qquad (1.23)$$

Comparing (1.22) and (1.23), approximately we obtain

$$A_0 = \sqrt{S_0};\ A_1 = 0;\ A_2 = \frac{S_2}{2\sqrt{S_0}}.$$

The dependence of average/mean value $<u_p>$ from z is called discriminatory characteristic. It usually possesses central symmetry, $<u_p(0)> = 0$, so that Function $<u_p>$ can be expanded in series/row according to degrees of z at point z=0:

$$<u_p(z, U_{вх}, П_{ш})> = K_p(U_{вх}, П_{ш})\, z + K_2(U_{вх}, П_{ш})\, z^2 + \cdots \qquad (1.24)$$

Value

$$K_p(U_{вх}, П_{ш}) = \left(\frac{d<u_p>}{dz}\right)_{z=0} \qquad (1.25)$$

is called the transmission factor of discriminator. In general case K_p it depends both on the value of signal and on noise level at the input of radio receiver.

If signal, in turn, is low and does not remain constant, but it is the random function of the time (usually this function in

comparison with the noise at the input can be considered slowly changing), then K_p will be also the random function of time.

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In this case we come to the system with the random transmission factor and, where this does not contradict the problem of analysis, we resort to the second (supplementary) averaging of transmission factor on the ensemble of the realizations of input signal, determining this correlation coefficient

$$K_p = \left(\frac{d < u_p (U_{вх}, \Pi_{вх}) >}{dU_{гх}} \right)_{x=0}. \qquad (1.26)$$

In the majority of the cases the interference can be considered stationary. Coefficient K_p with that fixed/recorded $U_{вх}$ decreases with increase/growth $\Pi_{вх}$. A considerable increase in the relation noise/signal can lead to so large a decrease K_p, that the system seemingly is broken and tracking ceases.

Let us return to dependence $K_p(U_{вх})$. The reason for this dependence consists in the fact that input value x (angle, frequency, time lag, etc.) in its physical nature differs from in terms of the high-frequency of the signals, which come the antenna systems (input computers) of radio installations. Thus, the antennas of system ASN enters not angle γ, but the high-frequency oscillations where this

angle is coded in some parameters of these oscillations; the input of system APCh it enters not frequency (or voltage/stress, proportional to it), but the sinusoidal oscillations where the value, after which is realized the tracking, enters under the sign of sine.

Coefficient K_p enters by factor into general/common/total transmission factor. Dependence $K(U_{вх})$ leads to uncontrollable changes in the transmission factor of servo system, which is inadmissible. Therefore in the measuring elements/cells are taken special measures for the exception/elimination of this dependence.

In those systems where the information about the monitored value is not included in the signal amplitude (for example, in the systems ASD, APCh), are utilized amplitude limiters. Where it is not possible to use them (for example, in the system ASN with the conical scanning), are established/installed the effective systems of the automatic gain control (ARU), making it possible to weaken this dependence to the acceptable limits.

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The stabilization of transmission factor succeeds in achieving, beginning from certain minimum level of input signal $U^*_{вх}$. Lower than this level coefficient of the transfer of measuring device/equipment

vanishes with unlimited decrease U_{BX}, which leads finally to the cessation of tracking (to interrupting system).

Value U^*_{BX} must lie/rest higher than that level, on which begins a considerable decrease in the transmission factor from the effect of input noise. As the illustration of the described positions can serve the discriminatory characteristics, obtained for different measuring elements/cells in different ratios noise/signal.

The curves of Fig. 1.7a are the discriminatory characteristics of balance type FM discriminator obtained by calculation [38] to input of which is supplied the sine wave of different frequencies and white noise. It is here marked:

$\langle u_{OTH} \rangle$ - the ratio of the average/mean value of output voltage/stress to the amplitude of the voltage of signal on the first duct/contour;

$\rho = \sigma^2 / P_c$ - ratio noise/signal

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_1 \sigma_2 R_{1,2};$$

P_c - the power of signal;

σ_1^2, σ_2^2 - dispersion of voltages/stresses on the I and II

ducts/contours of discriminator;

$R_{1,2}$ - coefficient of the cross correlation of noise edge stresses; β=kQ, k - coupling coefficient of ducts/contours, Q - their quality.

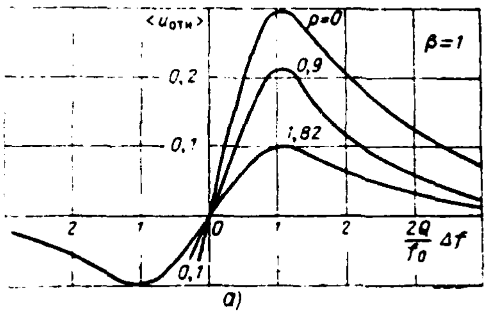

Fig. 1.7a).

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The experimental characteristics [1] of the average/mean value of voltage/stress for the frequency discriminator on the detuned circuits during the supplying to the input of Doppler signal (the "white" noise, passed through the narrow-band filter) and broadband noise are given in Fig. 1.7b.

FOOTNOTE [1]. They are obtained by S. S. Rudenko. ENDFOOTNOTE.

Here ρ - relation of effective disturbing voltages and signal on the input of discriminator.

Analogous experimental curves for the temporary/time discriminator are given in Fig. 1.7c. Here $\rho = \sqrt{2}\,\sigma_п/U_и$, $U_и$ - pulse amplitude, and $\sigma_п$ - dispersion of noise.

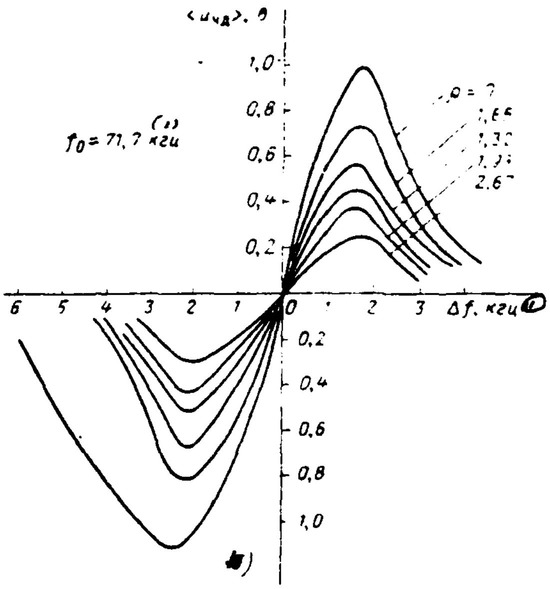

Fig. 1.7b.

Key: (1). kHz.

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Constructed according to these data dependences of relative transmission factor on the relation noise/signal are given in Fig. 1.8. Curves 1, 2, 3 here relate to Fig. 1.7a, b, c respectively, which confirm the predicted positions.

The work of automatic system in the design conditions

flows/occurs/lasts in the sufficiently large signal-to-noise ratio when K_p can be considered virtually depending either on $U_{вх}$ or from $\Pi_{вх}$.

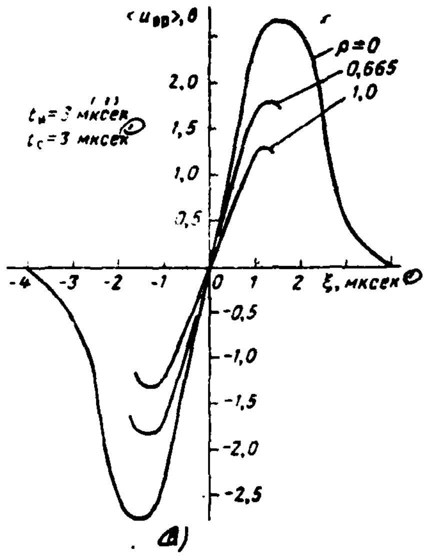

Fig. 1.7c.

Key: (1). µs.

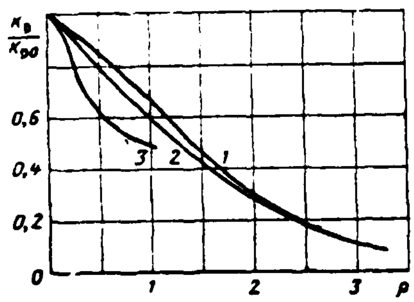

Fig. 1.8.
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In this case for the low divergences from z=0:

$$u_{\text{п}} = K_1 z + A\xi(t),$$

where $K_1 = \left(\frac{d < u_p >}{dz}\right)_{z=0}$ - constant value, which does not depend on $U_{вх}$ and $S_{п}$.

The respectively equivalent schematic of discriminator takes the simple form Fig. 1.9. In the general case this diagram is wrong and it is necessary to use expression (1.20) or resolution into:

$$\begin{aligned} u_p(t) &= K_p(S_п, U_{вх})\, z + K_2(S, U_{вх})\, z^2 + \\ &+ \cdots + (A_0 + A_1 z + A_2 z^2 + \cdots)\, \xi(t) = \\ &= A_0 \xi(t) + [K_1(S, U_{вх}) + A_1 \xi(t)]\, z + [K_2 + A_2 \xi(t)]\, z^2 + \cdots \end{aligned} \quad (1.27)$$

Latter/last representation coincides with the statistically equivalent filter, examined in [5], where is proposed the formula of the determination of the parameters of filter.

Thus the radio engineering servo systems, which are located under the effect of outside interferences, during the analysis must be related to the class of nonlinear systems with the random transmission factor whose mathematical expectation depends on the parameters noise/signal also of the parameters of input signal. The nonlinear character of dependence $u_p = \varphi(z)$ leads to the possibility of the loss of signal by tracking system.

Disruption/separation occurs in the case when disagreement/mismatch z falls outside the boundaries of characteristic $\varphi(z)$ (+-Δ in Fig. 1.10).

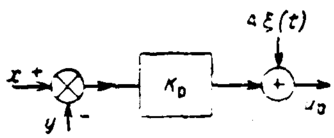

Fig. 1.9.

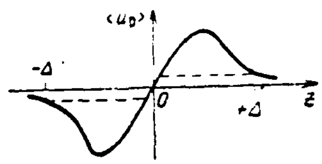

Fig. 1.10.

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The target of control in this case is broken and output value y changes as a result of the system of energy stored up in the separate elements/cells, and also under the effect of the noises, which penetrate to the output of measuring element/cell. It is possible that under the action of these noises the inequality $|z|<\Delta$ will be retained for a certain period of time and system again will pass into the mode/conditions of tracking. However, the probability of this event, as a rule, is small, and after the disruption/separation of tracking the system after certain time, adjusted previously, it is converted into the search mode which is organized by the fact or another method.

Since useful signal always is accompanied by noises, the probability of disrupting/separating the tracking is always different from zero. For the noises of high level this probability increases.

Probability of the disruption/separation of accompaniment the higher, the more rapidly changes the monitored value x. For the integrated systems, in which it is a difference in the complete input value of servo system and measured on board the flight vehicle, the probability of disrupting/separating the accompaniment under the equal conditions is always less than in simple system.

The disruption/separation of tracking is random event. It it is possible to rate/estimate by different statistical characteristics. Most frequently is utilized the mean time of accompaniment T_{cp}, by which it is understood average/mean statistical time from the moment/torque of the beginning of tracking to the moment/torque when z(t) reaches (any) one boundary value +-Δ. The mean time of accompaniment depends on the noise level, disagreement/mismatch, which exists in the system at moment/torque t=0, and also on the parameters of servo system.

The phenomenon of disruption/separation is examined in [1]. In connection with the problem of aggregation us it will interest dependence T_{cp} on the steady (initial) disagreement/mismatch z_0 to

system. Without stopping on the analysis of theoretical dependences, let us note that from increase of z_0 T_{cp} it falls, the form of dependence $z_0(T_{cp})$ in many respects it is determined by the characteristics of the subsequent elements of system.

Fig. 1.11 illustrates this position. Are here given calculated curves [1] for the mean time of accompaniment T_{cp} to functions $K_и$ (s^{-1}), found for the system Fig. 1.12a, that consists of the integrating element/cell [with the transmission factor $K_и$ (s^{-1})] and the measuring element/cell whose characteristic takes the form of the straight line in the section +-Δ with inclination/slope $K_{иу}=0,1$ V/deg, to input of which is given the signal, which is changed with constant velocity, $\dot{x}_0$.

FOOTNOTE [1]. They are obtained by Yu. G. Borisov. ENDFOOTNOTE.

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In the system, which has astaticism of the 1st order, is established/installed velocity error $z_0=\dot{x}_0/K_иK_{иу}$. During the supplying to the input of the additive noise of the specific level are observed the disruptions/separations of tracking, moreover at each selected value $K_и$ the time of accompaniment the greater, the less $\dot{x}_0$, i.e. the less the steady disagreement/mismatch z_0. Let us note that for

each assigned value $\dot{x}_0$ there is an optimum value of equivalent band $\Delta F_э = K_п K_ш / 2$ of the system, in which T_{cp} has the maximum. This maximum the less, the more $\dot{x}_0$ and the higher the noise level.

Fig. 1.13a, b gives experimental results on the investigation of the characteristics of the disruption/separation of tracking. They relate to the system ASD with astaticism of the 1st (Fig. 1.13a) and 2nd (Fig. 1.13b) orders in which the characteristic of temporary/time discriminator took the form Fig. 1.7c, moreover to the input of system were supplied the impulses/momenta/pulses, modulated in the amplitude by noise (amplitude noise of target) with the exponential correlation function of type (1.4).

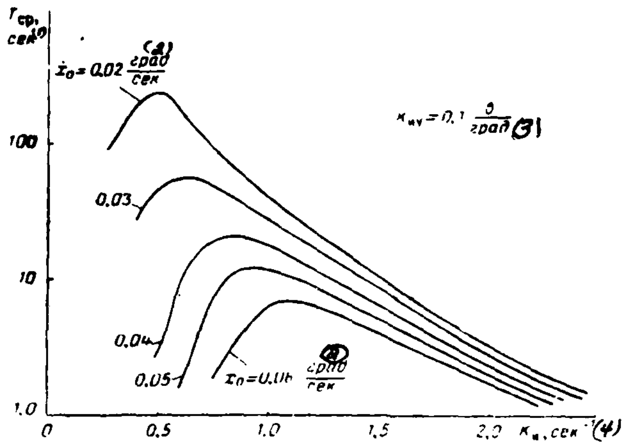

Fig. 1.11.

Key: (1). s. (2). deg/s. (3). deg. (4). s^{-1}.

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For the system Fig. 1.13a input signal takes the form of the linearly increasing function of time (speed $\dot{\varepsilon}_{вх}$), while for the system Fig. 1.13b - the function, which increases of the quadratic law (acceleration $\ddot{\varepsilon}_{вх}$). This provided the constant steady following error (in the absence of interferences), equal respectively: $\dot{\varepsilon}_{вх}/K_v$ and $\ddot{\varepsilon}_{вх}/K_a$. Along the axis of abscissas in Fig. 1.13 is deposited/postponed the ratio of the rms amplitude pulse envelope at the input of amplitude detector $U_{вхф}$ to the rms value of fun $\sigma_п$ at the same input:

$q_0=U_{\text{нэф}}/\sigma_{\text{п}}$. Curves are taken for different coefficients of correlation of interference α at values $t_{\text{вх}}$, of those corresponding to the speed of 500 m/s for the diagram in Fig. 1.13a, and $\bar{t}_{\text{вх}}$, corresponding to acceleration a_0=5 m/s^2.

For the development/detection of the effect which can be achieved/reached as a result of aggregation into the system it was introduced the compensating signal from ISD, determined rate of change in the input signal with certain error. For example, if as ISD was utilized the meter of airspeed, then its errors were received similar: fluctuation errors - normal stationary process with the correlation function of type (1.4), where $\sigma_{m\text{п}}$=3 m/s, $\alpha_m=1/6$ s^{-1}; and "constant" wind v=30 m/s.

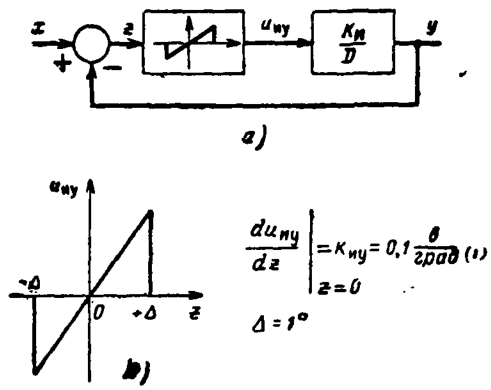

Fig. 1.12.

Key: (1). deg.

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For the unintegrated system from astatism of the 1st order with input signals indicated above optimum transmission factor was equal to 40 s^{-1}, for the analogous integrated system - 2 s^{-1}, which corresponded to the decrease of the limiting value of relation signal/noise $q_{0пр}$ 2.5 times. (By asymptotic relation signal/noise $q_{0пр}$ it is understood the value q_0, at which time T_{cp} is equal to the given one).

For the system with astaticism of the 2nd order due to the

analogous aggregation, optimum transmission factor changed from 2 s^{-2} to 0.4 s^{-2}, which ensured decrease $q_{опт}$ approximately/exemplarily in 1.5 times.

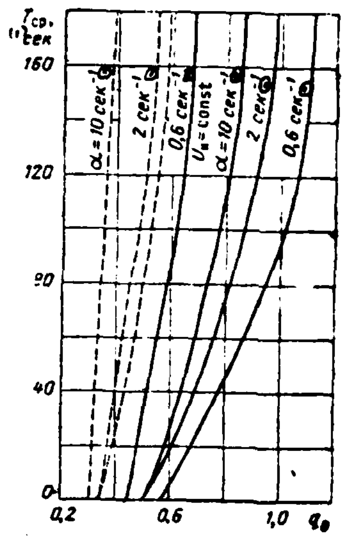

Fig. 1.13a.

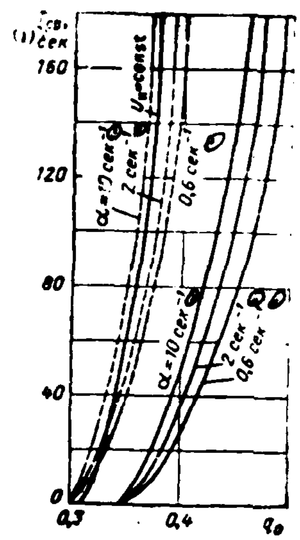

Fig. 1.13b.

Fig. 1.13a. --- --- simple system $(K_{\cdot} - 42.5\ s^{-1})$; - - - - integrated system $(K_{\cdot}-2\ s^{-1})$.

Key: (1). s.

Fig. 1.13b. --- --- --- simple system $(K_a = 2.4\ s^{-2})$; - - - - integrated system $(K_a = 0.42\ s^{-2})$.

Key: (1). s.

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The following important special feature/peculiarity of the servo radio engineering systems consists of the possibility of the loss of accompaniment as a result of the cessation of the entrance of radio signals for the input, caused by the prolonged fadings of signals or by any other reasons. Since the aperture of the characteristic of measuring element/cell is comparatively small, measured coordinate for the time when radio signals do not act, it can change so, that upon the new appearance of a radio signal the disagreement/mismatch will leave for limiting aperture $\pm\Delta$ of discriminatory characteristic and tracking it will not be restored/reduced: system must pass into the search.

For weakening of this effect it is necessary to increase the order of astaticism of system, which is frequently difficultly and in many instances does not lead to the desired result.

The radio engineering servo system with astaticism of the 1st order possesses on the position, the 2nd - on the speed. Let us explain, which this indicates. In the case of the disappearance of radio signal output potential of measuring element/cell becomes equal to zero. Therefore in the system with astaticism of the 1st order will be established/installed zero voltage on the input of integrator

and after the end of transient process output value will stop to equal input value at the moment of the disappearance of radio signal and it will remain constant/invariable subsequently (memory on the position).

In the system with astaticism of the 2nd order upon the disappearance of voltage signal on the input of the 1st integrator is set by equal to zero, and at its output (or the input of the 2nd integrator) - it will be preserved by such such as it was at the moment of disappearance. This means that output value after the end of transient processes will change linear in the time at a velocity, equal to that such as had input value at the moment of disappearance (velocity memory). If during the fading of radio signals rate of change in the input value does not remain constant, then for this system disagreement/mismatch can exceed the limits of the aperture of discriminatory characteristic, which will lead to the loss of tracking.

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In the integrated system the effect appears considerably weaker, since into the radio system continuously is introduced the information about the input value with ISD, so that the disagreement/mismatch between the actual value of input value and the

value, introduced from ISD, remains small.

The loss of tracking can occur in this case because ISD can put out information about the actual value of input value with the error which in certain cases can be accumulated.

Subsequently we will again touch these questions.

§1.2. Gyroscopic meters.

In the complex meters of coordinates, mainly, angles and their derivatives, as the self-contained sensors extensively are used the gyroscopes of different types.

1. Displacement gyroscope because of the presence of two framework has three degrees of freedom. The axis of its rotor does not change orientation in the inertial space. With the angular displacements of foundation, on which is established/installed the gyroscope, arises relative motion of the rotor and framework. The displacement of axes is converted into electrical signal, proportional to the value of the angular displacement of foundation.

2. Rate gyroscope has one framework motion by which is limited by elastic spring; attachment point of spring is extended beyond axis

of framework. The angular deflection of the output axis, perpendicular to the spin axis of rotor, is proportional the angular motion of base.

3. Gyroscope, which integrates speed (integrating gyroscope), with respect to device/equipment is identical to rate gyroscope with the difference that elastic limitation is replaced by viscous damping of output axis. The output signal of gyroscope is proportional to integral of the angular velocity of input axis.

4. Gyro horizon presents displacement gyroscope, equipped with device/equipment for rotation of rotational axis to position of local vertical line. As the means of the correction of the position of axis can be utilized the pendulum. The latter is held by force of gravity in the direction of local vertical line.

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The work of the system of correction is reduced to the imposition on the gyroscope of necessary in the value and the direction of the moment which makes it necessary to precess gyroscope to the assigned position.

5. Stabilized platform is foundation on which is

arranged/located one either two position or two rate gyroscopes in Cardan suspensions.

Gyroscopes serve for measuring the angles of deflection of platform relative to the assigned position (or their derivatives). The signals of the gyroscopes through the engine act on the axis of the gimbal suspensions so that the platform proves to be that stabilized in the inertial space. Usually are utilized two gyroscopes. Therefore it is possible to unload system from the action of the external exciting moments/torques.

There are still several types of gyroscopic sensors, including making it possible to measure angular accelerations [19].

The errors of gyroscopic sensors depend on the design (the moment of momentum of rotor, the type of suspension, the method of the removal of signal, the thoroughness of balancing/trimming, etc.) and the conditions in which they work. The determination of the statistical characteristics of the errors indicated in the function of so larger a quantity of parameters is virtually impossible. At the same time for calculation and engineering the complex meters of coordinates it is necessary to have at least the approximate data about the errors of sensors.

An error in the position or integrating gyroscope, the represented difference in the actual position of its output axis of relatively given one in the inertial space, can be represented in the form of the sum:

$$\Delta\psi_r = \Delta\psi_0 + \Delta\psi_y + \Delta\psi_\varphi + \Delta\psi_c, \qquad (1.28)$$

where $\Delta\psi_0$ - error in the initial installation/setting up (exhibition); $\Delta\psi_y$ - error, caused by the drift/care of the axis of gyroscope; $\Delta\psi_\varphi$ - error, caused by the displacement of flight vehicle and by the rotation of the Earth; $\Delta\psi_c$ - error in the system of removal and data transmission about the position of the axis of gyroscope.

The error of exhibition $\Delta\psi_0$ can be determined only experimentally.

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Being value, random from one experiment to the next, in each this experiment it is constant and its examination in the majority of the cases is not of interest.

However, it is necessary to keep in mind that the error of the initial exhibition of the axis of gyroscope leads to the appearance of connections/communications between channels of stabilization.

Component $\Delta\psi_y$ is usually the fundamental parameter, which characterizes the quality of gyroscope.

The main reason for the gyroscope drifts is the presence of moments of friction $M_T(t)$ and the imbalance of masses relative to the internal framework of suspension $M_p(t)$:

$$M_B(t) = M_T(t) + M_p(t), \qquad (1.29)$$

where $M_B(t)$ - exciting moment/torque.

Moment of friction $M_T(t)$ it is possible to present in the form two components: by the constant, which characterizes the inequality of the moments/torques of bearing friction during the rotation into one and other side, and by random component, caused by the imperfection of bearings.

Constant component for this gyroscope is random variable from one sample/specimen to the next and in the integrated systems of role does not play.

Random component, as is shown experiment, is the stationary function of time and its correlation function can be approximated by the expression

$$R_M(\tau) = \sigma_M^2 e^{-\alpha_M |\tau|} \qquad (1.30)$$

where σ_M^2 - dispersion of moment/torque $M_B(t)$; α_M - the parameter of correlation.

In this case coefficients α_M and σ_M must in each case be determined experimentally on the dynamic stand and under the conditions, close to the real ones.

Position and that integrates gyroscopes with respect to the exciting moments/torques are the integrating elements/cells; therefore the drift/care

$$-\beta_y = \int_0^t \frac{M_B(\tau)}{H} d\tau. \qquad (1.31)$$

where H - the moment of momentum of rotor.

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The gyroscope drift, caused by the constant component of moment of friction M_{T0}, is equal to:

$$\Delta\psi_{T0}(t)=\frac{M_{T0}}{H}t, \qquad (1.32)$$

where random for the party/batch of gyroscopes is value M_{T0} and, therefore, the dispersion of the drift/care

$$\sigma^2_{\psi 0}=\frac{\sigma^2_{T0}}{H^2}t^2. \qquad (1.33)$$

Dispersion of fluctuation component of angle $\Delta\psi_y$ is also the function of time and grows/rises according to the law:

$$\sigma^2_{\psi y}(t)=\frac{2\sigma^2_M}{\alpha_M H^2_M}\left[t-\frac{1}{\alpha_M}(1-e^{-\alpha_M t})\right]. \qquad (1.34)$$

During the calculation of the errors of integrated systems it is possible to utilize a method of converting the transient fluctuations into the quasi-stationary ones. The essence of method lies in the fact that in the limited intervals of time in which they are interested in the errors of complex, they represent the correlation function of the errors of gyroscope in the form:

$$R_{\text{экв}}(\tau)=\sigma^2_{\psi y}(t_к)\,e^{-\alpha_M|\tau|}, \qquad (1.35)$$

while $\sigma^2_{\psi y}(t_к)$ they determine for the moment/torque of time $t_к$ interesting from (1.34).

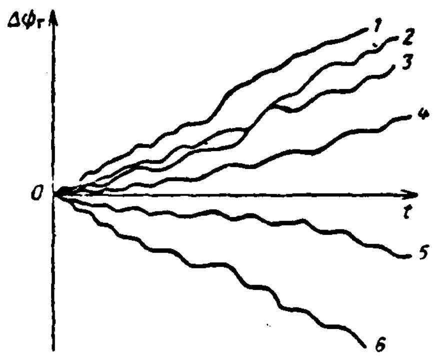

Fig. 1.14.

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The accuracy of method proves to be the higher, the more slowly changes $\sigma^2_{\phi y}(t_K)$. It is build-up/growth of dispersion is substantially lower than the time of setting process in the complex meter.

The character of the dependence of the instantaneous values of the errors of gyroscope from the time for different copies is represented in Fig. 1.14. Fluctuation component despite the fact that it is less than "regular" drifts/cares, can be decisive. As it will be shown subsequently, a linear-increasing error of gyroscope can completely be compensated due to the work of the radio engineering servo system.

Since the axis of gyrorotor is stabilized in the inertial space, the rotation of the Earth leads to its continuous divergence relative to the local vertical line (Fig. 1.15) with the angular velocity

$$\omega_{\psi} = \omega_3 \cos \varphi_M. \quad (1.36)$$

where ω_3 - angular velocity of rotation of the Earth (15°/h or 0.25°/min); φ_m - latitude of place.

Forward motion of flight vehicle also causes the misalignment of rotor relative to local vertical line. Let us assume that the flight vehicle moves at a rate of w at height/altitude H_c along the arc of the great circle (Fig. 1.16). Then the angular deflection velocity of gyroscope from the vertical line in the plane, passing through the longitudinal axis of flight vehicle,

$$\omega_{\gamma} = \frac{w}{R_3 + H_c}, \quad (1.37)$$

where R_3 - radius of the Earth (6371 km).

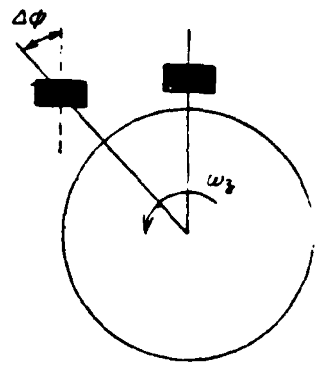

Fig. 1.15.

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In flight with any course $\psi_з(t)$ the misalignment of gyroscope in the plane, passing through the longitudinal axis of flight vehicle, occurs with the angular velocity:

$$\omega_{\varphi}(t)=\omega_з\cos\varphi(t)\sin\psi_з(t)+\frac{w(t)}{R_з+H_c(t)}. \qquad (1.38)$$

The misalignment of gyroscope in the plane, passing through the transverse axis of flight vehicle, is determined by the projection of the horizontal component of angular rate of rotation of the Earth to the direction of the axis indicated and it is equal

$$\omega_{cn}(t)=\omega_з\cos\varphi(t)\cos\psi_з(t). \qquad (1.39)$$

As can be seen from the given formulas, free displacement

gyroscope for purposes of stabilization can be utilized only during the short-term flights up to comparatively small distances. Where these conditions are not satisfied, must be utilized the correction of gyroscope. Correction can, be realized, for example, with the aid of the pendulum. In the integrated systems, such, as the goniometrical channels of homing systems, the correction of the position of the axis of gyroscope can be realized due to the signals of radio engineering meters.

The error of removal and transfer of signals from the gyroscope to the actuating element also it is possible to represent as being constant component $\Delta\psi_{c0}$, caused by imprecise initial installation/setting up of the data-transmission system, and random $\Delta\psi_{cc}$.

The reasons for the appearance of the latter can be different. Thus, if as the device/equipment of removal is utilized potentiometer, then output signal becomes discrete/digital and, therefore, appears the error, which it is accepted to name "quantization noise".

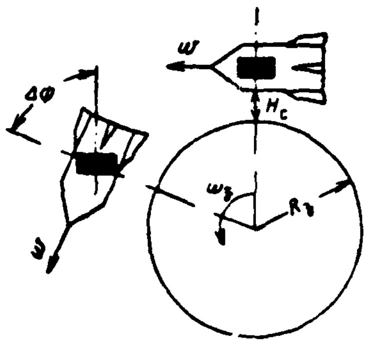

Fig. 1.16.

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If in the circuit of the transmission of signal are located reducers, signal is distorted additionally due to the phenomena of stagnation and gap.

Are possible the dynamic errors, caused by the inertness of the transmission system of signals.

Stagnation and discrete/digital character of the removal of data lead to the fact that the errors are not only the functions of time, but also measured angle

$$\Delta\psi_{cc} = f(t; \psi). \qquad (1.40)$$

Therefore the determination of their statistical characteristics is complex problem.

The errors of the rate gyroscopes, caused by imbalance and friction in the suspension, differ in the statistical characteristics from the errors of the integrating and displacement gyroscopes.

The equation of motion of the rate gyroscope takes form [19]:

$$J\frac{d^2\psi}{dt^2}+h_d\frac{d\psi}{dt}+K_n\psi=H\omega_c, \qquad (1.41)$$

where J - moment of inertia relative to the framework; ψ - angle of rotation (output signal); $h_d\frac{d\psi}{dt}$ — moment/torque of damper; $K_n\psi$ — moment/torque of spring; $H\omega_c$ - gyroscopic moment/torque; ω_c — angular velocity of the rotation of measuring axis. In the steady-state mode/conditions

$$K_{np}\psi=H\omega_c, \qquad (1.42)$$

whence

$$\psi=\frac{H\omega}{K_{np}}. \qquad (1.43)$$

Imbalance and bearing friction can be considered as random changes ω_c $(\Delta\omega_c)$ and statistical characteristics of the errors of output of signal to define as the result of the effect of random disturbances on the oscillating circuit/member. In this case, however, it is necessary to keep in mind that equation (1.41) is correct for small angles ψ.

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At the large angles the measured angle

$$\psi_{изм}=\frac{H\omega_c}{K_{np}}\cos\psi,$$

but

$$\Delta\psi_{\text{изм}} = \frac{H\omega_c}{K_{\text{пр}}}(1-\cos\psi). \qquad (1.44)$$

For decreasing the dynamic errors of the rate gyroscope should be chosen its natural frequency

$$\omega_0 = \sqrt{\frac{K_{\text{пр}}}{J}}$$

it is higher than the frequencies of the spectrum of input signal.

However, an excessive increase ω_0 can lead to the appearance of errors, caused by the vibrations of the housing of flight vehicle.

The errors of the rate gyroscope, the Earth caused by rotation by the displacement of flight vehicle, can be determined according to formulas (1.38) and (1.39).

Friction in the axes of gyroscope can be identified with the white noise, which affects its input. It is known [3] that the noises, which have the uniform spectrum, after the passage through the oscillating circuit/member have the correlation function:

$$R(\tau) = \sigma_{\text{ш}}^2 e^{-a_{\text{ш}}|\tau|}\left(\cos\omega_0\tau + \frac{a_{\text{ш}}}{\omega_0}\sin\omega_0\tau\right). \qquad (1.45)$$

It is easy to find the connection/communication of the parameters of correlation function $\sigma_{\text{ш}}$, $a_{\text{ш}}$, ω_0 with the parameters of gyroscope J, h_∂, $K_{\text{пр}}$, H. But the value of spectral density (or $\sigma_{\text{ш}}$) must be

determined experimentally.

Thus, the errors of gyroscopes can be represented in the form:

$$\Delta\psi_i = \Delta\psi_0 + \sum_{i=1}^{n} b_i t^i + \Delta\psi_c(t), \qquad (1.46)$$

where b_i — constant coefficients; $\Delta\psi_c$ — random stationary or transient (depending on the type of gyroscope) noise.

§1.3. Accelerometers.

Accelerometer - instrument, intended for measuring the linear or angular acceleration. The linear accelerations of flight vehicle are measured with the help of the linear accelerometers which are divided into the axial ones (Fig. 1.17) and the pendulum ones (Fig. 1.18).

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The operating principle of axial accelerometer [16] is based on what longitudinal acceleration a_x creates force of F, equalized by counteracting force of the spring, proportional to the linear shift of mass x, i.e.,

$$ma_x = K_{np} x,$$

where K_{np} — constant of spring.

The equation of motion of accelerometer has the same character, as (1.41).

The operating principle of pendulum accelerometer is illustrated by Fig. 1.18.

Free pendulum is set in the direction of that resulting of the forces acting on it; therefore this pendulum can be used as the meter of the resulting acceleration, caused by the forces applied to the flight vehicle.

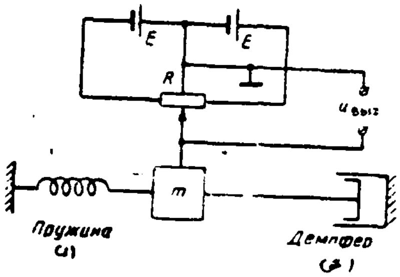

Fig. 1.17.

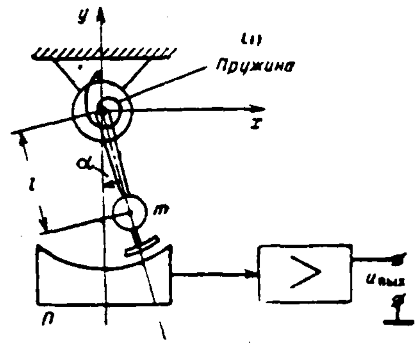

Fig. 1.18.

Fig. 1.17. Key: (1). Spring. (2). Damper.

Fig. 1.18. Key: (1). Spring.

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Pendulum with a mass of m and arm l is suspended/hung from the axis of damper and can be deflected relative to the sensor P, which records angles of deflection α. The sensor signals, proportional to angle α, are amplified and are supplied to the input of meter.

Pendulum stops, when begins the equality of counteracting force of spring and forces, applied to the mass. In this simplest form the accelerometer of longitudinal accelerations due to the action of forces of gravity. However, the actions of these forces it is easy to compensate [16].

If one assumes that the angle of deflection is small (it composes several minutes), then the equation of pendulum accelerometer will coincide with (1.41).

The operating principle of the accelerometer, which measures the angular velocity (for example, the speed of bank), is illustrated by Fig. 1.19. The flywheel rotating on the axis is fastened with the spring.

The motion of axis relative to flywheel with the angular acceleration ω will cause the elastic deformation of spring to the angle α and the moment of the elastic forces

$$M_{\text{п}} = K_{\text{пр}}\alpha,$$

where $K_{\text{пр}}$ — constant spring; it will be applied to the flywheel.

Flywheel will begin motion with angular acceleration $\dot{\omega}_{\text{м}}$. The

equation of motion of the flywheel

$$J_{M}\dot{\omega}_{M} = M_{\Pi},$$

where J_{M} — moment of inertia.

Angle α will cease to change at that moment/torque when

$$\dot{\omega} = \dot{\omega}_{N}.$$

The errors of accelerometers are caused by a number of factors and consist of two components:

- its own error of accelerometer;

- error, caused by the divergence of its axes relative to the local horizon and the assigned longitudinal orientation (error of the installation/setting up of the coordinate axes in the horizontal plane).

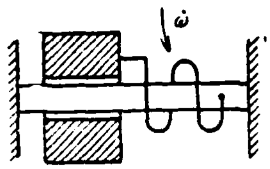

Fig. 1.19.

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Its own errors of accelerometer can be represented in the form

$$\Delta a = \Delta a_{\text{ш}} + \Delta a_{\text{д}} + \Delta a_{\text{с}}. \qquad (1.47)$$

where $\Delta a_{\text{ш}}$ — the "noises" of accelerometer; $\Delta a_{\text{д}}$ — dynamic error; $\Delta a_{\text{с}}$ — error of removal and transmission of signal from the accelerometer.

Random component of error ("noise") is caused by the series/row of the reasons:

- by friction at suspension points or in the guides (for the axial accelerometers);

- by disturbances/perturbations, caused by a change in the position of the center of mass of rocket or aircraft (due to the consumption/production/generation of fuel, jettisoning of loads, etc.) and varying the position of the center of mass of

accelerometer, relative to the center of mass of flight vehicle.

Dynamic errors are caused, first of all, by sudden wind gusts (by atmospheric turbulence) and by changes in the acceleration upon dispersal/acceleration and during braking of apparatus.

The components of errors, caused by changes in the flight conditions and by atmospheric turbulence, can be determined according to the formula

$$\Delta a_{R}(t)=[1-W_{a}(D)]W_{ca}(D)F(t), \qquad (1.48)$$

where $W_{a}(D)$ – transfer function of accelerometer; F(t) - the affecting along this axis force, applied to the flight vehicle; $W_{ca}(D)$ – transfer function of aircraft of the acceleration along the same axis.

The component of dynamic error, caused by the effect of atmospheric turbulence, is the stationary random function (if, of course, the transfer function of flying apparatus can be considered as stationary).

The dynamic errors, caused by a change of the engine thrust, should be represented as the determined functions of time.

All forms of random disturbances, except the gravitational

forces, connected with the change, which operate on the accelerometer, can be described analytically.

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So it is possible to consider that the frictional force, which affects the motion of the inertial mass of accelerometer, has approximately/exemplarily uniform spectrum in the limits of its passband. This assumption is to the certain degree hypothetical. In favor of this hypothesis speaks the fact that with the decrease of the mass of accelerometer the noises increase, and from an increase - they fall. Noises at the output of accelerometer will have the correlation function of form (1.45).

However, experiment is shown [11] that the damping factor even for the best types of dampers depending on the temperature in real conditions can be changed from 1 to 10. Therefore in the absence of heat stabilization the parameters of noises should be considered the function of temperature.

Apparently, the decisive role in the determination of the errors of the integrated system in which as sensing element is included the accelerometer, play the errors, caused by a change in the attitude of the measuring axis of accelerometer, for example due to the errors of

the stabilization system of platform. Thus, the divergence of the measuring axis of accelerometer of the angle $\Delta\varphi$ from the horizontal plane leads to the appearance of an error (Fig. 1.20):

$$\Delta a_x = a(1 - \cos\Delta\varphi) + g\sin\Delta\varphi, \qquad (1.49)$$

where g - acceleration of the Earth's gravity.

With a three-dimensional/space change of orienting the measuring coordinate system the signals in all three coordinates prove to be connected and the errors in any of the channels are the functions of the accelerations, which operate along all three axes. However, in the form of the smallness of the angles indicated it is possible to consider these errors as those not connected and the statistical characteristics of errors can be found, knowing the static characteristics of accelerations of the appropriate axes and the transfer function of stabilization system in the appropriate plane.

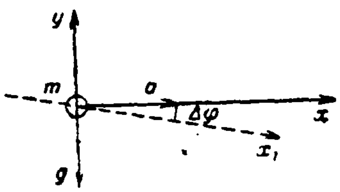

Fig. 1.20.

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The signals of the accelerometers before their introduction/input into the appropriate radio engineering servo systems must be converted to that coordinate system in which works this meter.

It is natural that to the unavoidable errors of the removal of signal they will be added the error of the device/equipment of translation. During the use as the latter of a digital computer these errors will be they have a character of "quantization noises", while during the use of analog units - continuous functions of time. In view of the nonlinearity of conversions the precision determination of the statistical characteristics of the errors of the device/equipment of the translation of coordinates represents, as a rule, problem not yielding to analytical solution and these characteristics they must it is located experimentally. At the same

time with respect to its own errors of acceleromters, in view of their smallness, the device/equipment of translation virtually always it is possible to linearize.

§1.4. Doppler meters of the vector of ground speed (DISS [Doppler speed-and-drift meter]).

DISS occupy important place among the navigation aids of aircraft and systems of control of pilotless objects. They are utilized for measuring the vector of ground speed whose components (longitudinal and transverse) after integration make it possible to determine that passed by flight vehicle path in earth-based coordinate system. In connection with the development of inertial navigation DISS they began extensively to be used as one of the meters of the integrated systems of navigation.

Thanks to the introduction of signals DISS to the inertial system of navigation succeeds in increasing substantially the general/common/total accuracy of the measurement of the navigational parameters of flight vehicle.

Assuming/setting the principle of construction DISS by the known (for example, see [31]), let us pause at the examination of the errors which have the vital importance during the use DISS as the

devices/equipment of the measurement of the components of vector or modulus/module of the vector of ground speed and drift angle.

These errors are composed of their own errors of the meter of Doppler frequency and errors in the course system, with the help of which is realized the resolving of vector of speed into two mutually perpendicular components.

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a) Errors in the meter of Doppler frequency.

Errors in this meter can be divided into the methodicals and the equipment ones.

The systematic errors are caused by the random character of the signals, reflected from the earth's surface, and by the dependence of echo strength from the angle of incidence in the radio waves on the terrestrial and especially sea surface.

It is known [31] that during irradiation of surface by the monochromatic signal of the section of the earth's surface, which has the final area (equal to the sectional area of the ray/beam of station by the plane of the earth/ground), the echo signal is the

narrow-band random process, the effective width of spectrum of which is connected with the beam width with the relationship/ratio:

$$\Delta F_{д}=F_{д0}\frac{\Delta\eta}{\sqrt{2}}\operatorname{tg}\eta_0, \tag{1.50}$$

where $\Delta\eta$ — beam width according to the half power in the radians; $F_{д0}$ — the Doppler effect of frequency; η_0 — angle between the axis of ray/beam and the sense of the vector of ground speed.

During the pulse radiation/emission the spectrum of the reflected signal can be considered as the superposition of the spectra, formed by reflection by each of the components of the spectrum of emitted signal.

During the motion of flight vehicle the conditions for the reflection of signal continuously are changed.

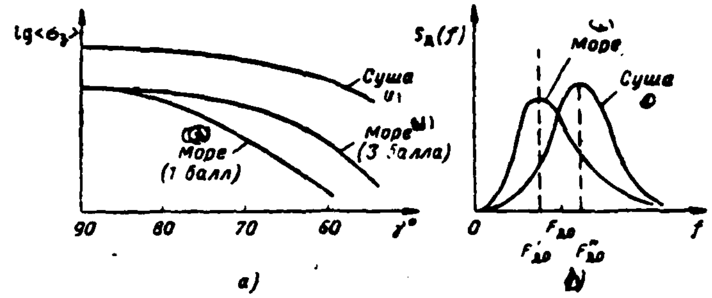

Fig. 1.21.

Key: (1). Land. (2). Sea. (3). Sea State 1. (4). Sea State 2.

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Furthermore does not remain constant and the cross section of target of the reflecting surface, which depends on the angle of incidence in the ray/beam γ (Fig. 1.21a). Therefore the spectrum of signal proves to be not always symmetrical relatively $F_{д0}$. In view of these reasons for Doppler the shift/shear of input signal DISS proves to be the random function of time whose average/mean values are displaced relatively $F_{д0}$, and dispersion $\sigma^2_{д0}$ depends on the character of surface. Virtually it is possible to count $\sigma_{д0} \approx (0{,}004-0{,}02) F_{д0}$. The spectrum of signal proves to be not always symmetrical relatively $F_{д0}$. The time of the correlation of the disturbances/perturbations indicated is changed in the very wide limits depending on the flight speed, character of locality, etc.

Furthermore, in flight above sea most reflecting surface frequently proves to be movable due to sea currents.

All enumerated disturbances/perturbations carry fluctuation character and errors caused by them they depend both on the statistical characteristics of disturbances/perturbations and method of processing signals in the station and, first of all, from the averaging time.

During the engineering calculations it is possible to consider that the correlation function of the error, caused by any of the enumerated above random disturbances, is determined only by the transfer function of the smoothing circuits, i.e., disturbance/perturbation itself can be identified with the white noise.

In DISS are utilized two types of devices/equipment for measuring the frequency $F_{зо}$ of the Doppler spectrum of the received signal:

- counters of "zeros";

- servo filters.

The block diagram of null indicator is given in Fig. 1.22.

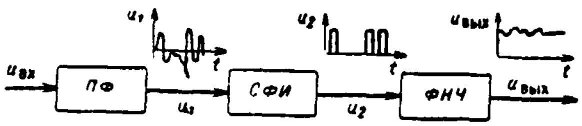

Fig. 1.22.

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Doppler signal $u_{вх}$ enters the band-pass filter (PF) with the passband from $F_{д\,min}-\Delta F_д$ to $F_{д\,max}+\Delta F_д$ and further it acts on the diagram of the formation of full-sized pulses (SFI) which works so that at its output appears the impulse/momentum/pulse each time when voltage/stress u_1 changes its sign from the negative to the positive. Full-sized pulses enter the smoothing circuit (most frequently single-unit filter RC). The spectral density of random component of surge voltage, caused by the chance of spacings between pulses, within the limits of the band of ripple filter is approximately/exemplarily constant. Therefore the correlation function of the error

$$R_д(\tau)=\frac{S_д(0)}{2T}e^{-\frac{|\tau|}{T}}, \qquad (1.51)$$

where T=RC.

The spectral density of fluctuations in the zero frequency is proportional to the width of the spectrum of Doppler signal can be to

approximation/approach determined according to the formula

$$S_{д}(0)=0{,}6\,\pi\Delta F_{д}, \qquad (1.52)$$

where $\Delta F_{д}$ in turn, is determined from (1.50).

System with servo filter is in detail examined in Chapter 6 (see pp. 202). Therefore here we will only note that the correlation function of error, caused by the random character of signal, will be determined by the transfer function of filter itself. If for the smoothing in the system is utilized single-section RC-filter, then correlation function will take the same form (1.51).

After using formulas (1.50), (1.52) and taking into account that

$$F_{д0}=\frac{2w}{\lambda_0}\cos\eta_0,$$

where λ_0 - transmitting wave RLS, it is not difficult to obtain formula for dispersive power of velocity measurement:

$$\frac{\sigma_w}{w}=\sqrt{\frac{0{,}025\,\Delta\eta\lambda_0\sin\eta_0}{\cos^2\eta_0 T w}}. \qquad (1.53)$$

If antenna system DISS is not stabilized, then with the banks and a change in the angles of attack appear supplementary equipment errors.

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Although when, on the aircraft, central gyro horizon is present, they

in the principle can be taken into consideration, during the error analysis DISS with them it is necessary to be counted.

With the decrease of signal-to-noise ratio of errors of DISS they increase. In this case together with fluctuation components appear the fixed bias in the measured values of Doppler frequency. Especially this is developed during the use of counters of "zeros". Therefore the latter are barely suitable for the integrated systems.

During the use of the servo noise filters of high levels also cause the constant components of errors. This is connected with the unavoidable asymmetry of the filters of the FM discriminators. It should be noted that with an increase in the noise intensity change the parameters of the FM discriminator (see Fig. 1.7b); therefore are changed the parameters of the correlation function of errors, in particular and α and ω_0 (1.45).

Together with the fluctuation errors to the meters of Doppler frequency are characteristic dynamic errors. Their steady values when $a_c = \text{const}$

$$\Delta w_{\text{д}} = \frac{a_c}{K}, \tag{1.54}$$

(1) где $K = \begin{cases} \frac{1}{T} \text{ (2) для счетчиков „нулей“;} \\ K_v \text{ — (3) коэффициент усиления контура для следящих фильтров.} \end{cases}$

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Key: (1). where. (2). for counters of "zeros". (3). factor of amplification of duct/contour for servo filters.

b) The errors of course system.

As a rule, for obtaining the necessary navigational parameters of DISS it is united with the course systems, with the help of which are realized the necessary transformations of coordinates. Most frequently as the course system is utilized the gyrocompass whose base composes position gyroscope; the errors of the latter were examined into S1.2.

During the accuracy analysis of the integrated system, which uses for the correction signals of DISS, it is necessary to consider also the errors of course systems (Chapter 6).

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S1.5. Meters of airspeed.

On each aircraft is installed the meter of airspeed, with the help of which aboard is measured the speed of aircraft relative to air medium.

Meter consists of the air-pressure head (PVD) and aneroid - speed indicator.

In the air position indicators instead of needle speed indicator is utilized the measuring device with the electrical output, which makes it possible to obtain air value of the speed in the form of voltage u_{yc}.

The meter of airspeed can be used in the integrated automatic systems as the source of information about the flight speed and work in different integrated systems, for example, together with the Doppler meter of ground speed. However, meter gives information about the airspeed, while for purposes of aggregation is desirable to have signals of the ground speed w, which differs from airspeed by magnitude of vector of wind u (Fig. 1.23).

The errors of the meter of airspeed are caused by the following reasons:

1. The impossibility of the account of the real density of air at flight altitude of aircraft caused the systematic error in the sensor. Is considered this indirectly, by the introduction/input of the corrections which are determined by the value of pressure and temperature Z to height/altitude [30, pp. 75].

2. By errors in transformation of diverging membranes/diaphragms of aneroid chambers to voltage, removed from wipers.

3. By atmospheric turbulence.

Let us consider the character of these errors.

At high speed of aircraft the drift angle α is usually small and with the steady course is changed comparatively slowly. Therefore it is possible to assume that with the help of the meter is determined the ground speed w with the slowly changing error Δw.

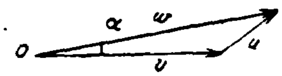

Fig. 1.23.

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Systematic and instrument errors $\Delta v_{м}$ are also the slowly changing functions of time. Thus, it is possible to consider that as a result of acting the reasons indicated true path airspeed w differs from the results of measuring the airspeed v for value $\Delta w + \Delta v_{м}$:

$$w = v + \Delta w + \Delta v_{м},$$

where Δw - function α, u, and v,

and $\Delta v_{м}$ – function of flight altitude, temperature and air density.

The errors, caused by turbulence, are changed in the time considerably more rapidly. A precise analysis of these errors is sufficiently complex, since it requires the account of the dynamic characteristics of aircraft. Therefore subsequently we will be bounded to the presentation of the physical causes for the onset of errors and results of proximate analysis.

At each point of airspace the velocity vector of air u it is possible to consider sum two components. One of them $u_{в}$ is the slowly varying function of time and is equal to the average/mean value (with the averaging in the sufficiently large segment of space), another $u_{фл}$ carries fluctuation character and is called turbulent component.

The number of investigations [37] established/installed, that atmospheric turbulence is isotropic. This means that for point of space, moving at a rate of u, the statistical characteristics of component $u_{фл}$ in any three mutually perpendicular directions in space $u_{фл\,x}$, $u_{фл\,y}$, $u_{фл\,z}$ can be considered identical, so that not one direction has advantages over another. Furthermore, turbulence of the atmosphere can be considered for the fast-moving aircraft also the stationary random function whose statistical characteristics do not depend on time, but they are the function only of the coordinates of isolated points.

Thus, in the investigations of eddying effect on the flight of aircraft the fluctuation field of the velocities can be assumed/set as by "that frozen" in the statistical sense.

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This means that the aircraft flies the distance for elongation/extent of which are still perceived temporary/time correlations between the fluctuations of velocity, it is so rapid that the field of wind velocity for this time does not change.

Such simplified treatment during which fluctuation component $\bar{u}_{\text{фл}}$ depends only on distance of r between two points of space and does not depend on time, will agree well with the experimental data [37] and frequently it is utilized for the analysis.

If vector $\bar{r}$ is directed along the velocity of aircraft, then for two points A and B of space, which are located at a distance of r, for describing turbulence it suffices to consider, as change tangential $u_{\text{фл т}}$ and normal $u_{\text{фл н}}$ fluctuation components; the third component $u_{\text{фл б}}$, perpendicular to two firsts, will have the same characteristics as $u_{\text{фл н}}$ (Fig. 1.24).

Correlation functions for the components indicated have the following expressions:

$$R_{\text{н}}(\tau)=\sigma_u^2\left(1-\frac{|\tau|}{2b}\right)e^{-\frac{|\tau|}{b}}, \qquad (1.55)$$

$$R_{\text{т}}(\tau)=\sigma_u^2\, e^{-\frac{|\tau|}{b}}. \qquad (1.56)$$

Here σ_u — the rms value of the velocity of turbulence - vary within the range of 0.4 m/s (flight in the static atmospheric conditions) to 2.7 m/s (strong turbulence);

$b=\frac{l}{\alpha_u}=\frac{L}{v}$ — parameter, which characterizes the time correlation; L - value, called the scale of turbulence and determined by the state of the atmosphere (by weather conditions, by flight altitude, etc.) - it varies within the limits of 200-1000 m (usually during calculations assume/set L=300 m).

The graphs/curves of standardized/normalized correlation functions (Fig. 1.25) show that relative value of the normal component of correlation function falls to level 0.1 with r/L=1.2, and tangential - 2.2.

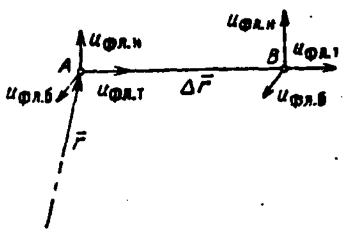

Fig. 1.24.

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With respect the spectral densities of turbulence will be located as the Fourier transform *of the correlation function:*

$$S_н(\omega)=\frac{\sigma_u^2}{\alpha_u}\frac{\alpha_u^2+3\omega^2}{\alpha_u^2+\omega^2}. \quad (1.57)$$

$$S_т(\omega)=\frac{2\alpha_u\sigma_u^2}{\alpha_u^2+\omega^2}. \quad (1.58)$$

Spectral density of normal components (Fig. 1.26) has a maximum at frequency $\omega=\alpha_u/\sqrt{3}$. For the evaluation/estimate of the width of the spectrum is convenient to use the concept of cut-off frequency $\omega_{гр}$, for which relation $S(\omega_{гр})/S(0)=0,1$.

Cut-off frequency for tangential and normal components is equal to respectively

$$\omega_{\text{гр т}} = 3\alpha_{\text{и}} = \frac{3v}{L} \cdot \frac{\text{рад}}{\text{сек}}; \quad \omega_{\text{гр н}} = \frac{5.3v}{L} \frac{\text{рад}}{\text{сек}}.$$

Key: (1) rad/s.

For v=300 m/s and scale of turbulence L=300 m, $\omega_{\text{гр т}}=3$ rad/s, $\omega_{\text{гр н}}=5.3$ rad/s.

Because of normal and side component of turbulence appear the oscillations of aircraft around the center of mass, in consequence of which are changed the angle of attack and slip angle. Therefore fluctuate components $u_{\text{фл н}}$ of the projection of velocity vector on the air-pressure head rigidly fastened with the airplane fuselage. These fluctuations can be considered distributed according to the normal law with rms value on the order of 4-5 m/s (for L=300 m and v=300 m/s).

Because of the presence of tangential component fluctuates the velocity of the flow incident to the aircraft. For the heavy aircraft these fluctuations $u_{\text{фл т}}$ are caught PVD and the spectrum of fluctuation (1.58) almost without the substantial changes is transferred to the output.

In the description of the dynamic properties of the meter of airspeed are considered the temporary/time time lag of waves in conduits/manifolds ($\tau_з$ ~~0.05-0.1 s), and also the inertial

properties of air cavities (τ_1), and the inertness of actuating element (indicator), and, $\tau_{\text{зу}}$ ~~0.05 s.

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Time constant τ_1 strongly depends on the diameter of conduits/manifolds and flight altitude. With d=6 mm on the earth/ground, τ_1=0.5 s; with H_c=10 km, τ_1=0.075 s.

If we do not consider time lag, then the output voltage/stress of speed indicator

$$u_{\text{ув}}=W_{\text{ус}}(D)v,$$

where

$$W_{\text{ус}}(D)=\frac{K_{\text{ус}}(0.5\tau_1 D+1)}{(\tau_1 D+1)(\tau_2 D+1)}$$

- transfer function of speed indicator; $K_{\text{ус}}$ — scale conversion factor of the units of the rate into unity of voltage/stress.

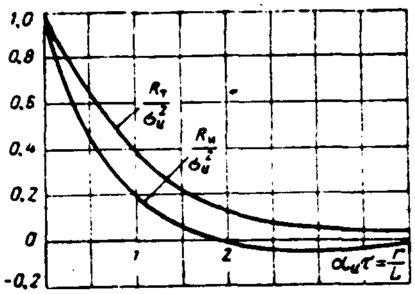

Fig. 1.25.

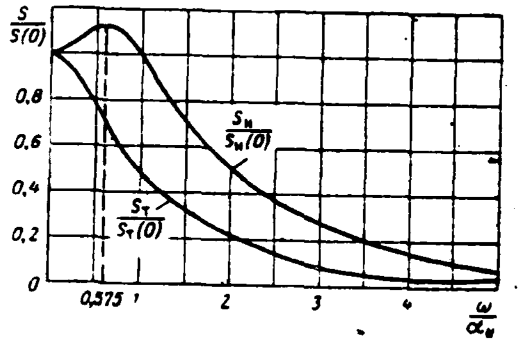

Fig. 1.26.

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The block diagram of the meter of airspeed is given in Fig. 1.27.

Since all components change comparatively slowly, inertial component/link with the time constant τ_1 (but in certain cases and by term $0.5\tau_1 D$ in the numerator and $\tau_1 D$ - in the denominator) frequently can be disregarded/neglected. In the simplest case the meter of the airspeed can be considered proportional component/link with the coefficient. In the simplest case the air speed meter can be condisered as a proportional element with coefficient K_{yc}.

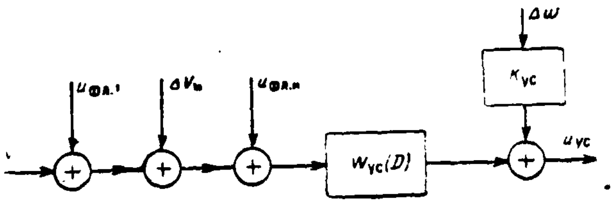

Fig. 1.27.

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Chapter 2.

PRINCIPLES OF AGGREGATION CLASSIFICATION OF INTEGRATED SYSTEMS.

In the chapter are examined the known principles of the aggregation of separate meters, and also the methods of the association of self-contained and nonautonomous radio engineering meters into the single integrated system. Is done the attempt to class integrated systems according to different signs/criteria.

In order to show, of what consist the generality and differences between the integrated systems and combined automatic control systems whose theory detailed at present, at first they are given very brief information along such systems.

§2.1. Brief information along the combined control systems.

The tendency to improve the dynamic properties of automatic control systems, to make these systems those more advanced, they led in recent years to the wide development of the ideas of the theory of invariance. Very fruitful proved to be the ideas of the theory

indicated in connection with the problems of aggregation.

This theory is occupied by the study of the possibilities of designing of the systems which would not react to the external disturbances/perturbations, applied to the controlled system (i.e. they were invariant with respect to them) and(without the dynamic errors) they ideally accurately reproduced input effect. The theory of invariance, which was conceived on the base of the idea, expressed by G. V. Shchipanov, was developed/processed subsequently by prominent Soviet researchers.

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Are at present formulated the basic condition/positions of this theory, systematized in A. I. Kukhtenko's work [7].

Let us consider some questions of the theory of invariance and will establish the connection/communication of invariant systems with the integrated systems.

For the realization of invariant system it is necessary (but it is insufficient)so that the system would have not less than two channels of propagation between the point of the application of external signal or external disturbance/perturbation and measuring

point of the value, with respect to which is provided the invariance (principle of two-channel construction, formulated by B. N. Petrov [8]).

Let us consider how is realized invariance in the case of using the combined control systems, i.e., the systems, in which the control is conducted both according to the principle of divergences and with respect to the principle of disturbances/perturbations. This is convenient to do based on the example of the system, represented in Fig. 2.1.

Disturbance/perturbation φ from the external source acts on the system (it is introduced into point C Fig. 2.1) and simultaneously along the compensating channel with transfer function $W_{\kappa}(D)$ enters certain point A of system (control on the disturbance/perturbation). The servo system [input x(t), output y(t)] it is supplemented by the channel of the compensation for input effect with the transfer function H(D) (position of the points of input of corrective commands A, B is not detented and can be changed, it is important only so that would be retained the order of alternation A with B). Mismatch in this system is found from expression [1]

$$\begin{aligned} z(t) = x(t) - y(t) = x(t) - W_1W_2W_3z(t) - \\ - W_1W_2W_3W_{\kappa}\varphi(t) - W_2W_3\varphi(t) + \\ + HW_3x(t) = (1 - HW_3)x(t) - Wz(t) - \\ - (WW_{\kappa} + W_2W_3)\varphi(t), \end{aligned} \quad (2.1)$$

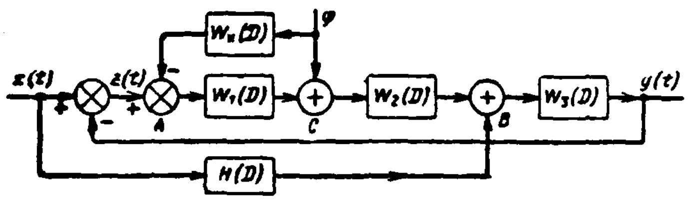

Fig. 2.1.

where

$$W = W(D) = W_1(D)\, W_2(D)\, W_3(D). \qquad (2.2)$$

FOOTNOTE [1]. Subsequently in many instances for the reduction of recording symbol D is omitted so that, for example, instead of W(D) is written/recorded simply by W. ENDFOOTNOTE.

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Hence we find

$$z(t) = \frac{(1 - HW_2)\, x(t)}{1 + W} - \frac{(W_2 W_3 - W W_к)\, \varphi(t)}{1 + W}. \qquad (2.3)$$

So that the system would be invariant with respect to the input effect x(t) and the interference φ(t), i.e., so that the disagreement/mismatch z(t) would not depend on these values, it is necessary and sufficient so that they would be implemented the following relationships/ratios:

$$1 - HW_2 = 0 \text{ и } W_2 W_3 - W W_к = 0, \qquad (2.4)$$

or

$$H = \frac{1}{W_2}; \quad W_к = \frac{W_2 W_3}{W} = \frac{1}{W_1}. \qquad (2.5)$$

Let us note that the obtained relationships/ratios correspond to z(t)=0 only for the zero initial conditions. If in the system are initial reserves of energy, then upon the inclusion/connection

appears the transient process during which $z(t) \neq 0$.

Satisfaction of the obtained conditions runs into the difficulty and it most frequently proves to be unrealizable. Actually/really, if W_1 and W_3 are the inertial components/links

$$W_1(D) = \frac{K_1}{T_1 D + 1}, \quad W_3(D) = \frac{K_3}{T_3 D + 1}. \qquad (2.6)$$

That the condition of invariance they lead to the equalities

$$H(D) = \frac{T_3 D + 1}{K_3}; \quad W_K(D) = \frac{T_1 D + 1}{K_1}. \qquad (2.7)$$

Thus, H(D) and $W_K(D)$ must implement the operation of the ideal boosting/forcing element/cell.

It is not difficult to see that if components/links W_1 and W_3 are the relations of polynomials whose degree of numerator is lower than the degree of denominator, then functions H(D), and $W_K(D)$ are simply physically unrealizable.

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However, these equalities it is possible to satisfy approximately. Thus, for instance, utilizing instead of the required ideal the real boosting/forcing component/link with the transfer function

$$H_p(D) = \frac{TD+1}{K_pTD+1} = \frac{TD+1}{\tau D+1}, \qquad (2.8)$$

where $K_p \ll 1, \tau = K_pT \ll T$, it is possible to obtain the system which will approach invariant.

With imprecise satisfaction of the conditions of invariance frequently it is possible to improve the accuracy of reproduction of input effect.

Thus, if $W_3(D)$ and $H_p(D)$ are expressed by the equalities

$$W_3(D) = \frac{K_3}{T_3D+1}, \quad H_p(D) = \frac{T_3D+1}{(\tau D+1)K_3}, \qquad (2.9)$$

moreover $\tau \ll T_3$, then for error z(t) (with $\varphi=0$) from (2.3) let us find

$$z(t) = \frac{\tau D}{[1+W(D)](\tau D+1)} x(t). \qquad (2.10)$$

If reference system had astaticism of the k order and

$$W(D) = \frac{P(D)}{D^kQ_1(D)}.$$

where polynomial $Q_1(D)$ does not have zero roots, then for this system

$$z(t)=\frac{x(t)}{1+W(D)}=\frac{D^{n}Q_1(D)}{P(D)+D^{k}Q_1(D)}x(t).$$

System obtained as a result of the introduction of component/link $H_p(D)$ has astaticism of the k+1 order, since for it

$$z(t)=\frac{\tau Dx(t)}{[1+W(D)](\tau D+1)}=\frac{\tau D^{k+1}Q_1(D)x(t)}{[P(D)+D^{k}Q(D)](\tau D+1)}. \tag{2.11}$$

Thus, the order of astaticism increased per unit, which can prove to be highly useful.

An increase in the order of astaticism per unit occurs also in the case when component/link $W_3(D)$ *contains* the components/links: integrating, inertial, the 2nd order and if rationally selected the structure of real compensating component/link $H_p(D)$.

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Let us note that the inaccuracy in satisfaction of the conditions of invariance, caused by the replacement of ideal components/links by real ones, cannot lead to the instability of system or to the fact that the system will cease to be rough.

Actually/really, taking into account that

$$y=x-z=\left[1-\frac{1-HW_1}{1+W}\right]x(t)-\frac{W_3W_1-WW_n}{1+W}\varphi(t), \tag{2.12}$$

and assuming that each of the transfer functions is a relation of two polynomials from D:

$$W(D)=\frac{P(D)}{Q(D)};\ W_{\text{з}}(D)\stackrel{\Delta}{=}\frac{P_{\text{з}}(D)}{Q_{\text{з}}(D)};\ H(D)=\frac{P_H(D)}{Q_H(D)}.$$

For the characteristic equation we will obtain

$$Q_H(D)[Q(D)+P(D)]_{D=\lambda}=0.$$

It decomposes into two factors. One of them $[Q+P]_{D=\lambda}=0$ is characteristic equation of reference system, but another $Q_H=0$ cannot give roots with the positive real parts, since component/link H(D) is stable.

Let us point out to one important fact, which makes with impossible ones the direct use of principle of the combined control for the radio systems in that form as this shown in Fig. 2.1. In any radio engineering servo system there does not exist explicitly the input signal x(t) and the disturbances/perturbations φ(t), that enters the system additively.

If, furthermore, system proves to be invariant with respect to the input effect, i.e., passes this effect without the distortions, then it passes to the output without weakening also of the interferences, which operate together with the input value. Specifically, these facts, characteristic for the radio engineering servo systems, make impossible the direct use of basic condition/positions of the theory of invariance.

However, position in principle is changed, if the input effect $x(t)$ or disturbance/perturbation $\varphi(t)$ becomes possible to measure with the help of any other devices/equipment.

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Then is created the possibility to introduce into the system the supplementary signals, proportional to input effect or to disturbance/perturbation.

Systems obtained thus will be complex, since their action is based on the use of several independent meters. Consequently, some ideas of the theory of invariance can be used to the integrated systems. However, the problems which appear during the aggregation, differ from those which solves the theory of invariance. Actually/really, during the aggregation each of the measured values contains interferences or errors. Signals must be mastered so that the resulting error would be minimum. This succeeds in doing because the transfer functions for the useful signals and the interferences are different, in contrast to the systems of the combined control.

Let us note also that during the use of several meters it

frequently proves to be that one of them (for example, radio engineering system) measures value itself, while others - its first-order or second derivative (velocity or acceleration). Consequently, into the automatic system it proves to be possible to introduce the signals, proportional to derivatives without the use of the differentiating circuits, moreover the conditions of the invariance of system on the input effect are satisfied automatically.

Subsequently these questions will be illuminated in detail.

§ 2.2. Functional diagrams of complex meters.

The sense of aggregation as this was indicated earlier, lies in the fact that to utilize information about one and the same or functionally connected values, obtained from different meters, for the purpose of an increase in the accuracy. If in complex are included radio engineering meters, simultaneously can be solved the problem of increasing their freedom from interference. In the signals, obtained from each of the meters, besides the useful information are contained also the measuring errors and interference.

It is necessary to come to light/detect/expose the physical essence of the gain which can be obtained, and the condition for its obtaining.

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For this let us consider the propagated methods of the association of meters - the diagram of compensation and diagram with the filters - and will show their identity. Furthermore, let us consider the diagram of aggregation with the radio engineering meter, in which the signal ISD is introduced into the radio engineering servo system.

A) The diagram of compensation.

The diagram of compensation [18] is very convenient for the development/detection of the physical essence of the gain which can be obtained as a result of the aggregation of two meters.

The signals of the first and second meters (Fig. 2.2) $x_1=x+\varphi_1$ and $x_2=x+\varphi_2$, containing the measured value x and interference φ_1 and φ_2, enter the subtractor VU, as a result of which the output signal of subtractor (at point A) is equal to difference $\varphi_1-\varphi_2$. Let, further, the interference spectra be located in different frequency domains, for example, the large part of the energy of interference φ_1 - in the low-frequency region, and interference spectrum φ_2 - is very wide-band (Fig. 2.3a). If now filter F is selected then so that it

with the minimum distortions would pass interference φ_1 and it as completely as possible suppressed interference φ_2, then at its output (at point B) will be almost completely reproduced interference φ_1. If it, further, are subtracted from signal x_1, then the signal obtained at the output will be very close to desired value of x.

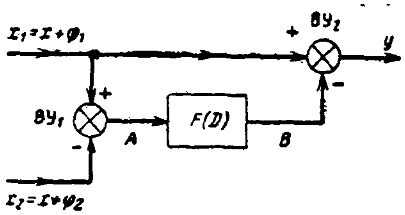

Fig. 2.2.

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Actually/really, if with the help of the filter it was possible to completely suppress interference φ_2 and to pass without the distortions interference φ_1, then signal at the output would be reproduced ideally accurately:

$$y = x + \varphi_1 - \varphi_1 = x.$$

In actuality this filtration cannot be carried out also in the output signal besides the required value x will be contained error z:

$$y = x + z.$$

The error z will be less, the stronger the difference in the spectral characteristics of interferences φ_1 and φ_2.

Let us find expression for the output signal. Signal at point B is equal to

$$x_B = F(D)(\varphi_1 - \varphi_2).$$

Consequently, the output signal

$$y = x_1 - x_B = x + \varphi_1 - (\varphi_1 - \varphi_2) F = x + (1 - F)\varphi_1 + F\varphi_2. \quad (2.13)$$

Thus, the error

$$z=(1-F)\varphi_1+F\varphi_2. \qquad (2.14)$$

According to the condition F accepted it must be the filter of low frequencies (Fig. 2.3b). Then filter with the characteristic 1-F will be high-pass filter.

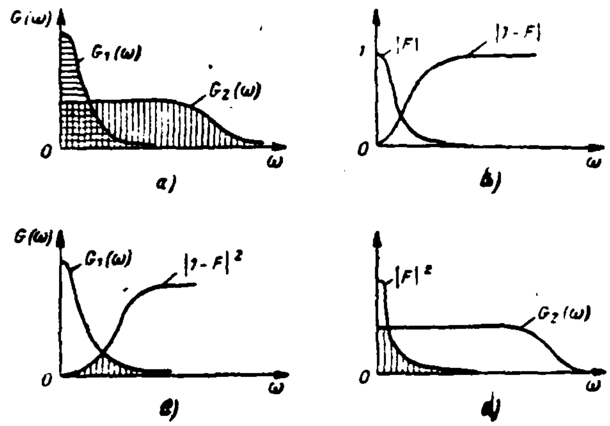

Fig. 2.3.

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To the output are passed the components of low frequencies φ_1, passed by high-pass filter 1-F, and also the components of high frequencies φ_2, which passed through the filter of low frequencies. If errors φ_1 and φ_2 are the stationary and statistically independent random functions of time with one-sided spectral densities of $G_1(\omega)$ and $G_2(\omega)$, then the variance of error of measurements will be equal to

$$\sigma_\varepsilon^2 = \frac{1}{2\pi}\int_0^\infty [G_1(\omega)|1-F(j\omega)|^2 + G_2(\omega)|F(j\omega)|^2]\,d\omega. \qquad (2.15)$$

As can be seen from Fig. 2.3c, d, this dispersion in the case when spectral densities "are spread" along the axis of frequencies, it is considerably less than the dispersion σ^2_1 and σ^2_2 the errors of the

input signals

$$\sigma_1^2=\frac{1}{2\pi}\int_0^\infty G(\omega)\,d\omega,\quad \sigma_2^2=\frac{1}{2\pi}\int_0^\infty G_1(\omega)\,d\omega.$$

Analogously it is possible to connect of compensation with three meters (Fig. 2.4) $x_1=x+\varphi_1$, $x_2=x+\varphi_2$ and $x_3=x+\varphi_3$.

For the output signals we will obtain

$$y_1=x+(1-F_1)\varphi_1+F_1\varphi_2=x+z_1, \tag{2.16}$$

$$y=x+(1-F_1)(1-F_2)\varphi_1+(1-F_2)F_1\varphi_2+F_2\varphi_3. \tag{2.17}$$

Weakening interference φ_3 can be achieved/reached when its energy is concentrated in the frequency region, different from that, where is concentrated energy of interference φ_1.

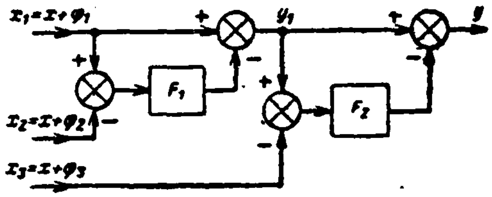

Fig. 2.4.

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From the given analysis of the diagram of compensation with two sources it is clear that it it is possible to present by equivalent diagram in Fig. 2.5, i.e., as the system, in which useful signal x is passed without the distortions, interference φ_2 - through filter with the transfer function F(D), and φ_1 - through the filter with the transfer function 1-F(D). If in this case filter F(D) passes lower frequencies, then filter 1-F(D), on the contrary, suppresses lower frequencies and passes upper. Therefore energy of interference φ_1 must be concentrated in the region of lower ones, and φ_2 - higher frequencies.

Let us consider a special case of such system whose self-contained meter can measure rate of change in the input value, i.e., value v=Dx with error φ_v. Then input x_1 of the diagram of compensation must precede integrator with transfer function K_n/D.

moreover $K_{\text{п}}=1\ s^{-1}$ (Fig. 2.6a). Consequently,

$$x_1=x+\varphi_1=\frac{K_{\text{п}}}{D}(v+\varphi_v)=x+\frac{\varphi_v}{D}.$$

The measuring error in this case, as before is expressed by formula (2.14), which let us record in the form

$$z=[1-F(D)]\frac{\varphi_v}{D}+F(D)\varphi_2. \qquad (2.18)$$

During the selection of filter F let us require so that in the output signal would not be contained the errors, caused by the constant errors in velocity measurement. For this must be satisfied the condition

$$\lim_{D\to 0}[1-F(D)]\frac{\varphi_{v0}}{D}=0, \qquad (2.19)$$

where φ_{v0} - constant component of error φ_v.

Otherwise the error of output signal will constant or grow/rise in the course of time. It is hence easy to establish/install the structure of filter F [12].

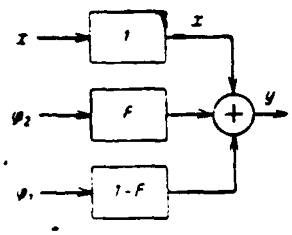

Fig. 2.5.

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Assuming that F(D) is the relation of two polynomials with $m \leqslant n$

$$F(D)=\frac{a_0D^m+\cdots+a_{m-1}D+a_m}{b_0D^n+\cdots+b_{n-1}D+b_n}, \qquad (2.20)$$

we will obtain

$$\frac{1-F(D)}{D}=\frac{1}{D}\,\frac{b_0D^n+\cdots+(b_{n-1}-a_{m-1})D+b_n-a_m}{b_0D^n+\cdots+b_{n-1}D+b_n}.$$

Condition (2.19) it is possible to satisfy, if only

$$b_n-a_m=0, \quad b_{n-1}-a_{m-1}=0. \qquad (2.21)$$

For the final establishment of the structure of filter let us take into account that the filter must suppress the high-frequency components of interference φ_2 (or at least it must not stress them), for which the degree of the polynomial of numerator must exceed the degree of the polynomial of denominator, i.e.

$$n \geqslant m+1. \qquad (2.22)$$

The enumerated conditions satisfies the polynomial of the minimum degree n=2, for which m=1:

$$F(D)=\frac{aD+b}{D^2+aD+b}. \qquad (2.23)$$

Output signal in this case will be

$$y = x + \frac{D\varphi_1}{D^2 + aD + b} + \frac{aD + b}{D^2 + aD + b}\varphi_2. \qquad (2.24)$$

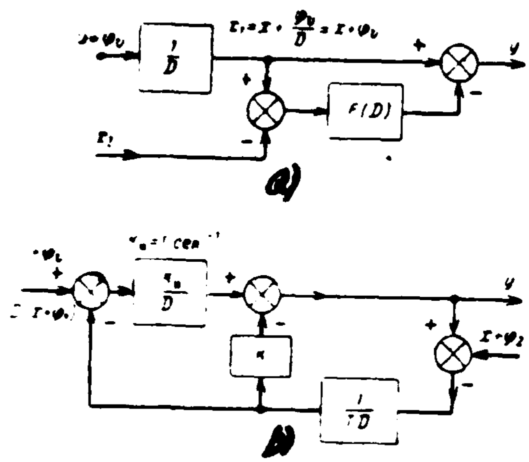

Fig. 2.6.

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Fig. 2.6b gives an example of the utilized circuit of connection of the results of measuring two sources examined. For it

$$y = x + \frac{TD\varphi_v}{M(D)} + \frac{KD+1}{M(D)}\varphi_2, \qquad (2.25)$$

where

$$M(D) = TD^2 + KD + 1,$$

which coincides with expression (2.24), if we assume

$$T = \frac{1}{b}, \quad K = \frac{a}{b}.$$

If we for the diagram of compensation present requirement so that in the output signal was not contained errors, caused by the constant errors in the measurement not of velocity φ_v and positions $\varphi_1 = \varphi_{10}$, then instead of condition (2.19) let us find the lightened

condition

$$\lim_{D\to 0}[1-F(D)]\varphi_{10}=0. \qquad (2.26)$$

Hence let us find:

$$\lim_{D\to 0}\frac{b_0D^n+\cdots+(b_{n-1}-a_{m-1})D+b_n-a_m}{b_0D^n+\cdots+b_{n-1}D+b_n}\varphi_{10}=0.$$

for which is sufficient satisfaction only of one condition $b_n=a_m$. For the polynomial of minimum degree, which satisfies condition (2.22) n=1 (m=0). Then we obtain

$$F(D)=\frac{1}{TD+1}, \qquad (2.27)$$

where

$$T=\frac{b_{n-1}}{b_n}=\frac{b_{n-1}}{a_m}=\frac{b_0}{b_1}=\frac{b_0}{a_0}$$

and

$$y=x+\frac{TD\varphi_1}{TD+1}+\frac{\varphi_2}{TD+1}. \qquad (2.28)$$

Thus, the order of filter decreased per unit.

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B) The diagram of filtration.

The diagram of filtration possesses symmetry in processing of incoming information and all meters in it as "are equal" (in the diagram of the compensation for this symmetry was not observed). Let us give this diagram immediately for n meters [32] (Fig. 2.7a).

The results of measurements n of the signals, which contain the

measured value x, and errors (interference) $\varphi_1, \varphi_2, \ldots, \varphi_n$

$$x_1 = x + \varphi_1;\ x_2 = x + \varphi_2; \ldots;\ x_n = x + \varphi_n$$

after passage n of filters $\Phi_1, \ldots, \Phi_n$ are summarized. Output signal

$$y = \left[\sum_{i=1}^{n} \Phi_i(D)\right] x(t) + \sum_{i=1}^{n} \Phi_i(D) \varphi_i(t). \qquad (2.29)$$

So that the system would not introduce the dynamic errors (i.e. the errors, caused by transient processes in the filters) under any laws of change x, it is necessary and it suffices to fulfill the following relationship/ratio:

$$\sum_{i=1}^{n} \Phi_i(D) = 1. \qquad (2.30)$$

Under this condition

$$y = x + \sum_{i=1}^{n} \Phi_i(D) \varphi_i(t). \qquad (2.31)$$

For the system with two meters (n=2) condition (2.30) gives

$$y = x + \Phi_1\varphi_1 + (1 - \Phi_1)\varphi_2 = x + (1 - \Phi_2)\varphi_1 + \Phi_2\varphi_2. \qquad (2.32)$$

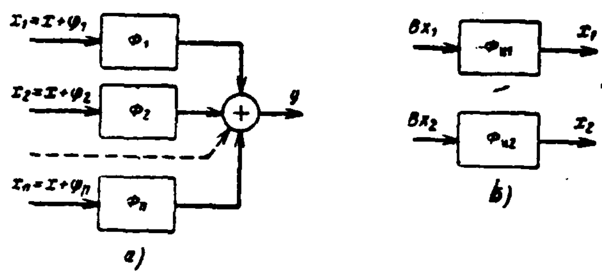

Fig. 2.7.

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Thus both systems (compensation and filtration) examined are completely equivalent, if we satisfy the condition:

$$F = 1 - \Phi_1 = \Phi_2 \tag{2.33}$$

All reasonings relative to physical treatment during the use of a diagram of compensation completely relate to the diagram with the filters.

In many instances it is necessary to consider that the primary meters possess dynamic errors, i.e., they themselves can be described by filters with transfer functions $\Phi_{и1}$ and $\Phi_{и2}$ [moreover $\Phi_{и1}(0) = 1 = \Phi_{и2}(0)$]. The input signals of the diagram of compensation for this case will be recorded in the form (Fig. 2.7b):

$$x_1 = \Phi_{и1} x + \varphi_1 = x + \Delta x_1 + \varphi_1,$$
$$x_2 = \Phi_{и2} x + \varphi_2 = x + \Delta x_2 + \varphi_2,$$

where the dynamic errors

$$\Delta x_1 = -(1 - \Phi_{и1})\, x,$$
$$\Delta x_2 = -(1 - \Phi_{и2})\, x.$$

Output signal of the diagram of compensation under this condition:

$$y = [1 - (1 - \Phi_{и1})(1 - F) - (1 - \Phi_{и2}) F]\, x + \\ + (1 - F)\varphi_1 + F\varphi_2. \qquad (2.34)$$

Consequently, besides the interferences in the output signal will be present the dynamic error

$$\Delta y_д = [(1 - \Phi_{и1})(1 - F) + (1 - \Phi_{и2}) F]\, x = \\ = (1 - F)\,\Delta x_1 + F\Delta x_2. \qquad (2.35)$$

The contraction of the band of the primary filters of meters ($\Phi_{и1}$, $\Phi_{и2}$) for the purpose of a decrease in the interference level φ_1 and φ_2 inevitably leads to an increase in the dynamic errors Δx_1 and Δx_2, which cannot be compensated and is caused the appropriate increase in dynamic error $\Delta y_д$ of the output signal of the diagram of compensation. The physical cause for this lies in the fact that dynamic errors of both meters lie/rest at one and the same frequency ranges.

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The schematics of integrated systems, constructed according to the principle of compensation or filtration, extensively are used in non-radiotechnical meters (for example, in the gyromagnetic compasses, ground-position indicators, etc.).

In connection with the radio engineering meters, which are most frequently servo, it is expedient to utilize another method of aggregation.

C) Diagram with the servo radio engineering meter.

Let us pass to the examination of the diagram of aggregation, in which one of the meters is the servo radio engineering system. If this system was linear, then it would be possible to be bounded to reference to the already examined diagrams. However, the special features/peculiarities of the radio engineering servo systems noted earlier make it necessary to resort to the more detailed description of such integrated systems. Furthermore, frequently during this aggregation as this will be clearly from the following, as the integrators and the ripple filters it is convenient to utilize the elements/cells, which functionally contain in the radio engineering system.

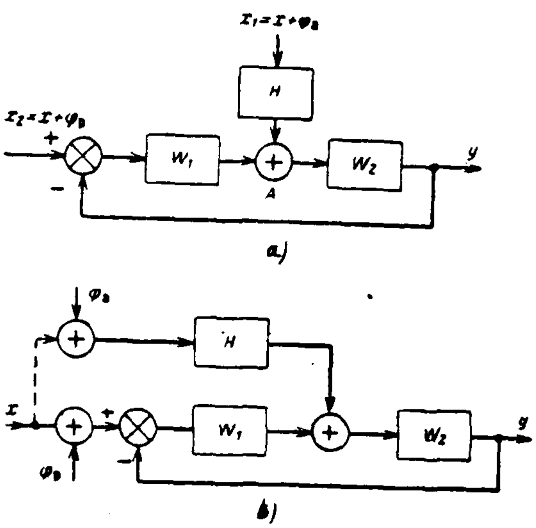

Fig. 2.8.

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Let the meter of proper motion determine the same value x, that also the radio engineering servo system, and possesses error φ_a. Radio engineering system we will represent in the form of two components/links with the transfer functions W_1 and W_2 (Fig. 2.8a). The measuring element of radio engineering system, which is here assumed/set by linear with the constant parameters, is included in component/link W_1. Component/link W_2 we will also consider linear. Let us assume that the input of radio engineering meter enters the additive mixture of signal x and noise φ_p:

$$x_1 = x + \varphi_p.$$

Usually noise φ_p is by sufficiently broadband in comparison with the band of system and width of the spectrum of error φ_a of self-contained meter. The signal of the self-contained meter x_1 is introduced additively into point A of the radio engineering system through the component/link with the transfer function H. For the measured value of y of value x in this case we will obtain

$$y=\frac{W+HW_2}{1+W}x+\frac{W}{1+W}\varphi_p+\frac{HW_2}{1+W}\varphi_a, \qquad (2.36)$$

where

$$W=W_1W_2.$$

So that in the output signal it would not be contained dynamic errors, must be performed the relationship/ratio

$$HW_2=1. \qquad (2.37)$$

Then

$$y=x+\frac{W}{1+W}\varphi_p+\frac{1}{1+W}\varphi_a. \qquad (2.38)$$

In the output signal is contained the measured value x and the errors, caused by interference φ_p and error of self-contained meter φ_a.

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If we suppose that

$$\frac{W}{1+W}=F=\Phi_p, \qquad (2.39)$$

first

$$1-F=\frac{1}{1+W}, \qquad (2.40)$$

and

$$y = x + (1 - F)\varphi_a + F\varphi_p =$$
$$= x + \Phi_p\varphi_p + (1 - \Phi_p)\varphi_a, \qquad (2.41)$$

and we come to the following conclusion: if radio engineering system is considered linear, then with satisfaction of condition (2.37) diagram in Fig. 2.8a is completely equivalent to the diagram of compensation or to diagram with the filters. [In this case condition (2.39) always it is possible to satisfy].

Error of the reproduction

$$z = x - y = -\frac{W}{1+W}\varphi_p - \frac{1}{1+W}\varphi_a.$$

The component of the error, caused by the interference of the radio engineering meter

$$z_p = -\Phi_p(D)\varphi_p(t) = -\frac{W(D)}{1+W(D)}\varphi_p(t), \qquad (2.42)$$

is located by the transmission of input error φ_p through the filter with the transfer function of the locked radio engineering system, while the component of the error of the self-contained meter

$$z_a = -\frac{1}{1+W}\varphi_a = -(1 - \Phi_p)\varphi_a \qquad (2.43)$$

is located as the result of the passage of interference φ_a through the filter with the transfer function of the error signal (Fig. 2.9).

Let us pause at condition (2.37). It is clear that it completely is equivalent to the condition of invariance (2.5). This will become especially demonstrative, if diagram in Fig. 2.8a is depicted in the

form Fig. 2.8b. In this case condition (2.37) can be satisfied even in such a case, when W_2 contains the integrating component/link. If condition (2.37) is not observed, then in the output signal will appear the supplementary dynamic error whose value will depend on how is great a difference in the real relationships/ratios from the conditions of invariance.

The representation of complex meters in the form, depicted in Fig. 2.8b, possibly when ISD measures the first or second derivatives of signal x.

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For this it suffices to include/connect component/link with the transfer function D or D^2 before the point of input of signal φ_k.

But if the conditions of invariance are satisfied and self-contained meter does not possess the dynamic error (i.e. $\Phi_{x1}=1$), then complex system also does not have dynamic error and the band of radio engineering system should be chosen only on the basis of the conditions of guaranteeing the minimum of interference errors (2.42) and (2.43).

Hence follows the important conclusion: complex system works so

that the systematic disagreement/mismatch at the output of the measuring element of radio engineering system, caused by dynamic error, is absent, and are created least favorable conditions for interruption of tracking. Certainly, this conclusion pertains to the idealized system. But also in the real integrated system the steady disagreement/mismatch is less than in the separately undertaken radio engineering system. Hence it follows that during the aggregation is obtained the supplementary gain in the freedom from interference in the sense that decreases the probability of disrupting/separating the tracking.

§ 2.3. Classification and the transfer functions of integrated systems.

As the basis of the classification of integrated systems with the radio engineering meter can be placed different signs/criteria.

It is considered advisable to lead the classification of the simplest systems with by two sources of information according to the following signs/criteria: to the completeness of information which can be obtained from the meter of proper motion (ISD), to the form of the operational connection/communication between the input signals of the servo radio engineering system and ISD, according to a number of utilized ISD, and also on the presence of the couplings between the separate meters.

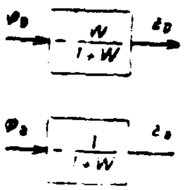

Fig. 2.9.

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A) System with the complete and incomplete information about the measured value.

By the first sign/criterion are distinguished two classes of the integrated systems which conditionally can be named thus:

- systems with the complete useful information;

- system with the partial (incomplete) information.

The systems in which ISD gives the same general/common/total information about the measured coordinate, that also the input signal of radio system, can be named systems with the perfect information. Let us clarify this by examples.

Let be realized the measurement of the distance between the ship K and coast radio beacon M. For simplicity we will assume that the ship moves to the beacon by M. Odny meter it is the range-only radar, which measures the distance D (Fig. 2.10a), by others - ISD - inertial system with the help of which it is possible to carry out a numeration of covered path $Д_а$. In the signal ISD is here contained information about that passed by the ship of path $Д_а$ and, consequently (with the known initial distance D_0), the same information, that in the input signal.

In this case

$$x_1 = x + \varphi_a = (Д_0 - Д_a) + \varphi_a = Д + \varphi_a, \qquad (2.44)$$

$$x_2 = x + \varphi_r = Д + \varphi_r. \qquad (2.45)$$

If, however, radio beacon M is established/installed on transfer table (for example, by friend ship M_R Fig. 2.10b), then as a result of dead reckoning we as before can obtain information only about path $Д_а$, the passed ship relative to the earth/ground (inertial space).

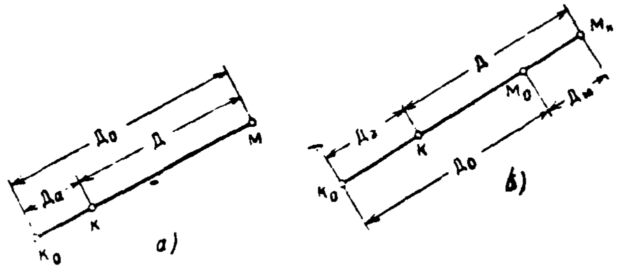

Fig. 2.10.

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The input signal of radio system and signal ISD will be recorded as follows:

$$x_1 = Д_a + \varphi_a, \qquad (2.46)$$

$$x_2 = Д + \varphi_1 = Д_0 - Д_a + Д_м + \varphi_1. \qquad (2.47)$$

composing paths $Д_м$, caused by the displacement/movement of movable radio beacon, cannot be taken into consideration by inertial system. Thus, during the aggregation in the sensor signals is contained all information about the measured value, and this system should be related to the system with the incomplete information.

Each meter puts out the measured values with the specific errors. However, during the determination of the class of system it is necessary that in these errors there would be no components, which contain the values, functionally connected with the components of the input signal of radio engineering meter.

Let us clarify this. Let us return for the given example. Signal ISD could be recorded in the form:

$$x_1 = Д_a + \varphi_a = Д_a + Д_M + (\varphi_a - Д_M) =$$
$$= Д_a + Д_M + \varphi'_a, \qquad (2.48)$$

where

$$\varphi'_a = \varphi_a - Д_M. \qquad (2.49)$$

During this recording is created the impression, that ISD gives perfect information about the useful coordinate (about a change in distance $Д = Д_0 + Д_M - Д_a$), but with error φ'_a and this system it was necessary to relate to the class of systems with the perfect information. However, this is not so, since here error φ'_a contains component $Д_M$, functionally connected with the input signal of the radio system (it is equal the same component $Д_M$, containing in the signal radio system). Therefore this system should be related to the class of systems with the incomplete information.

The functional diagram of system with the perfect information was examined above (page 70).

Let us pass to the examination of the special features/peculiarities of system with the incomplete information about the input value.

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These systems are encountered in the practice very frequently, namely in the cases of measuring the coordinates of the moving targets from the moving object.

Thus, the input value of the goniometrical coordinator (see Chapter of 4 pages 147) is angle $x_2=\gamma=\gamma_a+\gamma_\Pi$. ISD, established/installed on board the flight vehicle, can determine only component γ_a of common angle γ. It is analogous, in the measurement of time delay $t_{\text{вз}}$ (see page 180) ISD can determine its only that component, which corresponds to its own displacement/movement of flight vehicle relative to inertial reference system.

Thus, the input signals of integrated system with the incomplete information can be recorded thus:

$$x_1=x_a+\varphi_a, \qquad (2.50)$$

$$x_2=x+x_a+\varphi_\Gamma. \qquad (2.51)$$

Here input value x_2 of radio engineering meter contains supplementary component x by which no in the signal ISD. Let us note that coordinate x which is previously unknown, and is of greatest interest, since component x_a we actually" know". Therefore is correct to name coordinate x the useful information, to be determined with

the help of the integrated system, and x_a - by a priori information about the measured coordinate.

From the diagram in Fig. 2.11a it follows that output (measured) value in the integrated system:

$$y=\frac{W}{1+W}x+\frac{W+HW_2}{1+W}x_a+\frac{W}{1+W}\varphi_p+\frac{HW_2}{1+W}\varphi_a, \qquad (2.52)$$

where

$$W=W_1W_2.$$

In output value there are components, which depend respectively on x, x_a, φ_p and φ_a.

Let us record the error of reproduction, equal to a difference in desired value $x+x_a$ and measured value y:

$$z=x_a+x-y=\frac{1}{1+W}x+\frac{1-HW_2}{1+W}x_a-\frac{W}{1+W}\varphi_p-\frac{HW_2}{1+W}\varphi_a \qquad (2.53)$$

or

$$z=z_д+z_{да}+z_p+z_a. \qquad (2.54)$$

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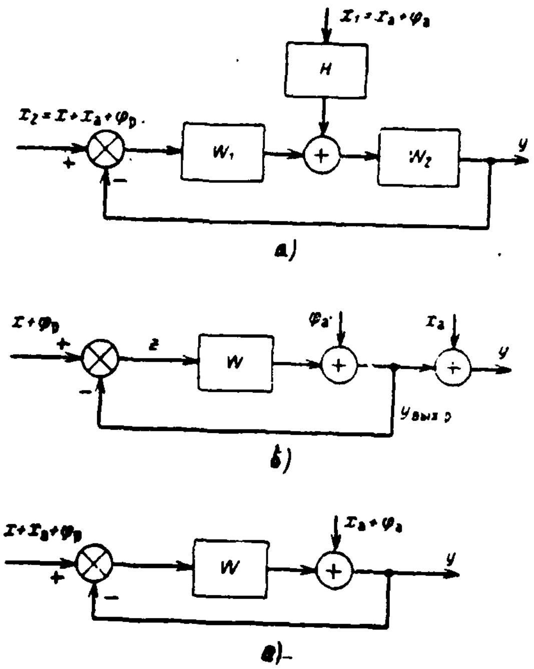

Fig. 2.11.

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Thus, the error of reproduction contains dynamic measuring error by useful component x:

$$z_{д}=\frac{1}{1+W}\,x=(1-\Phi_{г})\,x=\Phi_{\varepsilon}x \qquad (2.55)$$

- uncompensated remainder/residue of signal general/common/total for both meters x_a:

$$z_{дa}=\frac{1-HW_2}{1+W}\,x_a; \qquad (2.56)$$

- error, caused by an error in self-contained meter φ_a:

$$z_a = -\frac{HW_2}{1+W}\varphi_a; \qquad (2.57)$$

- the error, caused by the action of radio interference φ_p:

$$z_p = -\frac{W}{1+W}\varphi_p = -\Phi_p\varphi_p. \qquad (2.58)$$

Here $\Phi_p = W/1+W$ - transfer function of closed system;

$\Phi_s = 1 - \Phi_p$ - transfer function by mistake.

With satisfaction of the conditions of the invariance:

$$1 - HW_2 = 0, \text{ or } H = \frac{1}{W_2}, \qquad (2.59)$$

occurs the complete compensation for signal x_a general/common/total for both meters and error $x_{да} = 0$.

In this case

$$y = x_a + \frac{W}{1+W}x + \frac{W}{1+W}\varphi_p + \frac{1}{1+W}\varphi_a, \qquad (2.60)$$

i.e. the error of reproduction z (2.54) does not contain component $z_{да}$, and

$$z_a = -\frac{1}{1+W}\varphi_a. \qquad (2.61)$$

Hence it follows that with satisfaction of the condition of invariance the system will possess the dynamic error, caused by the action of not measured ISD and, therefore, uncompensated component of the input signal of radio system.

Thus, during the introduction of signal ISD radio engineering system actually realizes the measurements only by uncompensated component of x of common signal, since component x_a with satisfaction of the conditions of invariance completely is deducted from component x_a, containing in the signal ISD.

Let us clarify this with the help of the block diagrams (Fig. 2.11). Designating through $y_{\text{вых р}}$ difference $y-x_a$, from (2.60) we will obtain

$$y_{\text{вых р}}=y-x_a=\frac{W}{1+W}(x+\varphi_r)+\frac{1}{1+W}\varphi_a. \quad (2.62)$$

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To this equality corresponds the diagram in Fig. 2.11b, from which clearly is confirmed the expressed confirmation about the fact that the radio engineering system tracks only value x. For the demonstrative representation of the compensation effect for component x_a is conveniently the block diagram of integrated system represented also in the form Fig. 2.11c.

Interference φ_p in the integrated system passes through the filter of lower frequencies Φ_p, corresponding to the transfer function of closed system, and interferences φ_a — through the high-pass filter, which corresponds to transfer function by mistake. The spectral characteristics of interferences φ_a and φ_p usually are arranged/located in the different frequency ranges. Therefore for the explanation of that how it is obtained gain in the accuracy of reproduction of signal $x+x_a$, it is possible to utilize the same Fig. 2.3 as in the diagram of compensation. Difference lies in the fact that during the selection of the system characteristics it is necessary to consider dynamic error z_x, caused by the component of x

of input signal.

Rate of change in coordinate x is frequently considerably less than rate of change x_a. Actually/really, let us suppose that with the help of the radio engineering meter, for example radio compass or another direction finder, on the aircraft is realized the tracking the ship on which is established/installed the homing station (beacon).

The components of mutual angular displacements, caused by the motion of ship relative to the Earth, change considerably slower than the components of mutual displacements, caused by aircraft motion.

In this case dynamic error $z_д$, caused by action only comprising x of complete input signal $x+x_a$ of radio engineering system, will be less than in simple system. On the other hand, if rate of change in complete effect $x+x_a$ will be considerably more than the velocity of component x, the passband of complex system it is possible to select considerably narrower than the band of noncomplex. With the narrower passband the freedom from interference of integrated system will be above with identical dynamic errors $z_д$.

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By this is explained the usefulness of the use of a complex system [1].

FOOTNOTE [1]. We spoke here about the velocity for the convenience. In actuality one should speak about the more complicated characteristics, which describe input signals. Thus, if they are by chance and stationary, under "velocity" it is possible to understand the value, reciprocal of the time of correlation, etc. ENDFOOTNOTE.

Thus, in the systems with the incomplete information the selection of band depends not only on the spectral characteristics of interferences φ_a and φ_p, but also on the characteristics of useful, uncompensated component of x of input signal. Than "slower" is changed this component, the more effect can be achieved/reached as a result of aggregation.

Let us turn again to equalities (2.54), (2.56) and (2.57). From them it is evident that the conditions of invariance affect only the value of two components of accumulated error: $z_{да}$ and z_a. The error of reproduction of the component x and the error, caused by radio interferences, do not depend on that, are satisfied the conditions of invariance. Hence it follows that during the evaluation of the effect of an inaccuracy in satisfaction of these conditions on the properties of complex system it is possible to be bounded to the examination only of the two components of error, namely z_a and $z_{да}$,

clear rest.

B). Systems with the correction on the position, the velocity and the acceleration.

ISD make it possible to obtain the signals, proportional not only to the component of the input effect of radio system, but velocities Dx_a and accelerations D^2x_a. From that, which of these values is measured with the help of ISD it depends the character of the errors of integrated systems. Therefore it is considered advisable to distinguish systems by the sign/criterion indicated, after dividing them into:

- position (with the correction on the position), where with the help of ISD is measured component x_a, general/common/total for both meters;

- with the correction on the velocity where with the help of ISD is measured signal, proportional Dx_a;

- with the correction on the acceleration where with the help of ISD is measured the signal, proportional D^2x_a.

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In connection with this separation it is expedient to touch questions of the introduction/input of corrective commands and possibilities of satisfaction of the conditions of invariance.

Signals ISD are introduced into the fixed points of the schematic of the radio electronic system: the signal of acceleration - to the input of the second integrator (in this case radio engineering system it must possess astaticism of the 2nd order), the signal of velocity - to the input of the first integrator (if system - with astaticism of the 1st order), the signal of position - to the output of the latter/last integrating component/link or the input of system [1].

FOOTNOTE [1]. First is counted the integrator, closest to the output of system. ENDFOOTNOTE.

In this case to the maximum degree are utilized the elements/cells, general/common/total for both meters, since the correlation of the dimensionalities of the signals of self-contained and radio engineering meters is realized by elements/cells (integrators) of the latter, although in the principle it is possible this to do and with the help of the supplementary integrators and signal ISD to introduce

to the output of servo system. Structurally/constructurally this is it cannot be done either on the purely technical reasons or, for example, in the case of static system with the high-speed/high-velocity correction in view of the absence of integrators in the servo system. Then for the agreement it is necessary to resort to the introduction of supplementary integrators.

Example 1. Let there be the astatic servo radio engineering system whose block diagram is represented in Fig. 2.12, so that W_2 contains one integrating and inertial component/link, and all other components/links are contained in W_1. With ISD enters the signal of velocity of value x_n with certain error, i.e., $x_1 = Dx_n + \varphi_{n}$. At the output of radio engineering meter is signal $x + x_n$ and an interference φ_p. An example relates to the system with speed correction. Signal ISD can be fed:

- to point A, where in the radio engineering system is a signal, proportional to velocity (with an accuracy to dynamic errors), through "matching" component/link H(D);

- into point B, where in the radio system is a signal, proportional to the tracked value. In this case in the matching filter H(D) must be contained the integrator, which converts velocity Dx_n into value x_n.

It is not difficult to be convinced of the fact that the errors of complex meter do not depend on that, at what point is introduced into the following system the signal ISD. The point of introduction/input is chosen from the purely design considerations.

Let us consider the characteristics of meter on the assumption that the signal ISD is introduced at point A, as this is shown in Fig. 2.12a.

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Since filter H(D) affects components $z_{\text{ш}}$ and $z_{1\text{ш}}$, the passage of signal x examine we will not be. Since in the real system in component/link $W_2(D)$ can be contained nonlinearity, will possibly prove to be useful the preliminary filtration of signal x_1 in order to weaken/attenuate the interference suppression of signal in the nonlinear elements/cells. This is taken into consideration by filter $W_{\text{ф}}$ in the circuit ISD (for some types of meters, for example for the sensor of airspeed - page 49 - inertness it possesses meter itself).

We convert diagram to the form Fig. 2.12b, where

$$H_{\text{э}}=\frac{H(D)D}{T_{\text{ф}}D+1};\quad \varphi_{\text{э}}=\frac{\varphi_{\text{вв}}}{D}. \qquad (2.63)$$

Then formulas (2.50) and (2.61) for the error ISD and the uncompensated remainder/residue of general/common/total signal will take the form

$$z_a = \frac{H_e W_g}{1+W} \varphi_a. \qquad (2.64)$$

$$z_{\Delta a} = \frac{1 - H_e W_g}{1+W} x_a. \qquad (2.65)$$

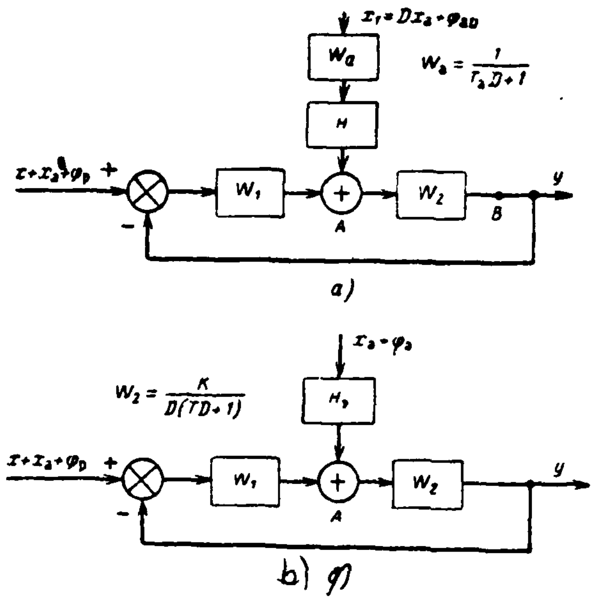

Fig. 2.12.

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The condition of invariance with W_2=K/TD+1 it is

$$H_э = \frac{1}{W_2} = \frac{D(TD+1)}{K}.$$

or, by taking into account (2.63), we will obtain

$$\frac{H(D)D}{T_a D + 1} = \frac{D(TD+1)}{K},$$

and

$$H(D) = \frac{(TD+1)(T_a D+1)}{K}. \qquad (2.66)$$

It is clear that the real circuit cannot have this transfer function. The condition of invariance (matching) can be satisfied

with the use of real elements/cells only in the case of the absence of inertness in component/link W_2 and the absence of filter $W_3 (T=T_3=0)$.

Then

$$H=\frac{1}{K} \qquad (2.67)$$

and the matching filter is a simply proportional component/link.

In the real case instead of the component/link with transfer function (2.66) should be taken the real component/link, which can have transfer function, for example the form

$$H_p(D)=\frac{(TD+1)(T_3D+1)}{M(D)K}, \qquad (2.68)$$

where $M(D)=\tau^2D^2+2\xi\tau D+1$ - certain quadratic polynomial. Then taking into account (2.63) instead of H_3 we obtain

$$H_{\phi p}=\frac{H_p(D)D}{T_3D+1}=\frac{D(TD+1)}{KM(D)} \qquad (2.69)$$

and the errors, caused by the nonfulfillment of the conditions of the invariance

$$z_{\phi p}=\frac{H_{\phi p}W_2}{1+W}\varphi_a+\frac{1}{1+W}\cdot\frac{1}{M(D)}\cdot\frac{\varphi_{a0}}{D}, \qquad (2.70)$$

$$z_{x\phi p}=\frac{1-H_{\phi p}W_2}{1+W}x_a+\frac{1}{1+W}\left[1-\frac{1}{M(D)}\right]x_a= =\frac{1}{1+W}\cdot\frac{\tau(\tau D+2\xi)}{M(D)}Dx_a. \qquad (2.71)$$

In the absence of filter W_3, M(D) it is degenerated into the linear binomial $\tau D+1$, and formula (2.79) will take the form

$$z_{x\phi p}=\frac{1}{1+W}\cdot\frac{\tau D}{\tau D+1}x_a. \qquad (2.72)$$

The given formulas show the following:

1. With respect to value x_{a} overall for two meters is raised per unit the order of astaticism.

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Thus, for the radio engineering meter with astaticism of the 1st order and transmission factor on velocity K_{v} steady error $z_{\text{н.уст}}=0$, if x_{a} is changed with the constant velocity. This error is equal to $\ddot{x}_{a}\tau/K_{v}$ in the absence of filter W_{a} and $2\xi\tau\ddot{x}_{a}/K_{v}$ in the presence of filter, if $\dot{x}_{a}$ is *changed with uniform acceleration* $\ddot{x}_{a}$. In the transient mode/conditions the error is found from formula (2.71) or (2.72). Let us recall that with precise satisfaction of the conditions of invariance always $z_{\text{н}a}=0$.

2. With respect to constant errors $\varphi_{a\text{v}}$ of ISD order of astaticism per unit is lower than order of astaticism of servo system. For the system with astaticism of the 1st order constant errors $\varphi_{a\text{v}}$ in velocity measurement are not chosen by system, i.e., if $\varphi_{a\text{v}}$ contains constant component $\varphi_{a\text{v}0}$, then this will bring in the steady-state mode/conditions to the constant error

$$z_{\text{н}0}=\frac{\varphi_{a\text{v}0}}{K_{v}} \qquad (2.73)$$

regardless of the fact, is there in the system supplementary filter

φ_a. In other words, with respect to value φ_a, the system is static.

So that this error would be absent, it is necessary to raise the order of astaticism, i.e., to introduce into the system supplementary integrator.

In the presence in φ_a, increasing in the time component, error z_{m0} in the steady-state mode/conditions will also grow/rise.

Let us again note that the same results can be obtained for the case when for the input of signal ISD is chosen point B.

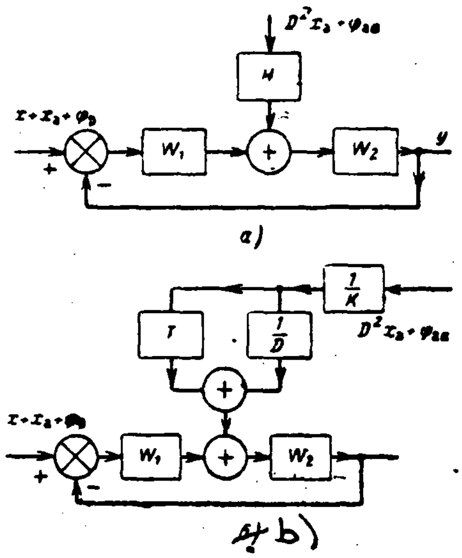

Fig. 2.13.

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Consequently, during the selection of the point of the introduction/input of corrective command should be been guided practical considerations, taking into account that during the transfer of the point of the introduction/input of the signal of high-speed/high-velocity correction to the output of system will be required the supplementary integrator, while during the introduction/input of signal into point A is utilized the integrator, available in the radio engineering servo system.

Example 2. Let us consider system with the correction on the acceleration (Fig. 2.13a), i.e., let us assume that ISD puts out the signal of acceleration with error φ_{aa}:

$$x_1 = D^2x_a + \varphi_{aa}. \qquad (2.74)$$

Let us replace the matching component/link H with equivalent ones H_s, by input of which it is supplied signal $x_a+\varphi_a$, so that

$$H_a = D^2H, \quad \varphi_a = \frac{\varphi_{aa}}{D^2}. \qquad (2.75)$$

Then the condition of invariance will take the form

$$H_a(D)W_2(D) = 1$$

or

$$H_a(D) = \frac{D(TD+1)}{K}, \qquad (2.76)$$

and the matching component/link

$$H = \frac{T}{K} + \frac{1}{KD} = \frac{1}{K}\left(T + \frac{1}{D}\right). \qquad (2.77)$$

Hence it follows that in contrast to the preceding/previous case satisfaction of the conditions of invariance does not produce fundamental difficulties and it can be carried out on the diagram in Fig. 2.13b).

Error, caused by error φ_{aa}.

$$z_a = -\frac{1}{1+W}\varphi_a = -\frac{1}{D^2(1+W)}\varphi_{aa}. \qquad (2.78)$$

If error φ_{aa} contains constant component, then so that it would not produce the errors in the steady-state mode/conditions, system must possess astaticism of the 3rd order.

In the system with astaticism of the 2nd order the constant component of error φ_{a0} will cause the constant error:

$$z_{a0}=\frac{\varphi_{aa0}}{K_a},$$

where K_a – transmission factor of system on the acceleration. Thus, with introduction of acceleration is simplified satisfaction of the conditions of invariance, but it can complicate the constant components of errors of ISD.

Laws governing the behavior of integrated systems in the steady-state modes/conditions, examined in the preceding/previous examples, carry general character.

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Let us assume that into the servo radio engineering system is introduced the k derivative of component x_a of proper motion with error φ_{ak}.

With imprecise satisfaction of the conditions of invariance the error, caused by component $x_a^{(k)}$, will not be equal to zero $(z_{\text{зар}}\neq 0)$. However, the order of astaticism with respect to this value will be per unity more than order n of astaticism of the radio electronic system (i.e. is equal to n+1).

The error, caused by error φ_{ak} in the steady-state mode/conditions, depends on that, are satisfied the conditions of invariance. Let us find, when in the steady-state mode/conditions is eliminated the action of errors φ_{ak} in ISD, if the latter are described by polynomial.

Let the signal of correction $x_1 = D^k x_a + \varphi_{ak}$ be introduced to the input of component/link $W_2(D)$, which contains k of the integrating components/links (Fig. 2.11a), and component/link $W_1(D)$ system contains l of integrating components/links (l+k=n). Then it is possible to record:

$$W_1(D) = \frac{P_1(D)}{D^l Q_1(D)}; \quad W_2(D) = \frac{P_2(D)}{D^k Q_2(D)}, \qquad (2.79)$$

where polynomials P_1, P_2, Q_1, Q_2 do not have zero roots. Then from (2.61) with HW=1 we find:

$$z_a = -\frac{\varphi_a}{1 + W_1 W_2} = -\frac{D^l Q_1 Q_2 \varphi_{ak}}{D^l D^k Q_1 Q_2 + P_1 P_2}. \qquad (2.80)$$

Assuming/setting $\varphi_{ak} = \varphi_{ak0}$, we will obtain in steady-state mode $\lim_{D \to 0} z_a = 0$ under condition l=1. If φ_{ak} increases according to the linear law, $\lim_{D \to 0} z_a = 0$ under condition l=2 and so forth.

Consequently, the order of astaticism of the component/link, which precedes the point of introduction/input, must be raised by value l=1, 2, ... in accordance with the law of a change in regular error φ_{ak}.

With position correction $\varphi_{ak}=\varphi_a$ and so that the error z_a in the steady-state mode/conditions would be absent, the system should possess astaticism of the 1st order with $\varphi_a=\varphi_{a0}$, astaticism of the 2nd order, if φ_a changes on the linear, etc.

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In conclusion let us note that the integrated system possesses the memory so that if the input radio engineering signal disappears, because of the input of the signal of self-contained meter the system continues to master introduced signal x_a and the accumulation of disagreement/mismatch occurs only due to the action of the component of x of input signal. The "Memorization" of the latter does not differ from the fact that occurs in the usual simple system.

C) systems with the use of several meters.

In some cases has the capability to utilize for the correction of radio system not one, but two ISD. Most exponential is the case of introduction to the astatic radio engineering system of signals from the speed/rate meters and acceleration.

In point A of system (Fig. 2.14a), where

$$W_2(D)=\frac{K}{D(TD+1)}, \qquad (2.81)$$

are introduced signals from meter 1

$$x_{1r} = Dx_a + \varphi_{av} \qquad (2.82)$$

and meter 2

$$x_{2a} = D^2 x_a + \varphi_{aa} \qquad (2.83)$$

through the matching components/links with the transfer functions $H_1(D)$ and $H_2(D)$ respectively.

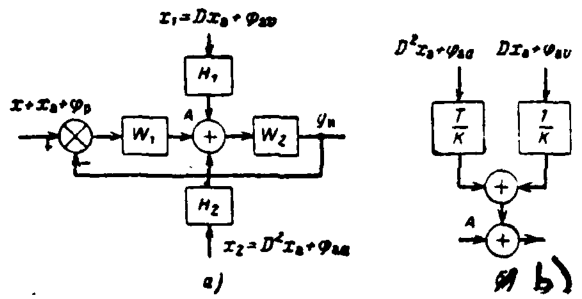

Fig. 2.14.

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Taking into account only the values, connected with reproduction x_{a}, φ_{av}, φ_{aa}, for appropriate components of errors we will obtain

$$z_{\text{да}}=\frac{1-[D^2H_2+DH_1]W_2}{1+W}x_{a},\qquad (2.84)$$

$$z_{a}=-\frac{(H_2\varphi_{aa}+H_1\varphi_{av})W_2}{1+W},\qquad (2.85)$$

where as before $W=W_1W_2$.

Invariance with respect to the signals of self-contained meters will occur, if $z_{\text{да}}=0$, i.e., when is satisfied the condition

$$(D^2H_2+DH_1)W_2=1.\qquad (2.86)$$

Taking into account expression (2.81), we come to equality

$$H_2D+H_1=\frac{1}{K}(TD+1)\qquad (2.87)$$

or

$$H_1=\frac{1}{K},\quad H_2=\frac{T}{K}.$$

Satisfaction of the conditions of invariance (2.87) does not produce difficulties and it can be realized, for example, as it is indicated in Fig. 2.14b.

With satisfaction of these conditions

$$z_a = -\frac{1}{K}\cdot\frac{\varphi_{av}+\varphi_{aa}T}{1+W}W_1. \qquad (2.88)$$

Hence it follows that constant component errors in the measurement velocities and acceleration, if component/link W_1 does not contain the integrating components/links, is produced the constant error in the steady-state mode/conditions

$$z_{a0} = \frac{\varphi_{av0}+\varphi_{aa0}T}{K_v},$$

where K_v — transmission factor of system with astaticism of the 1st order.

In order to do this error of equal to zero, it is necessary to increase the order of astaticism of system.

The evaluation/estimate of the advisability of using several meters can be given only as a result of the account of the statistical characteristics of the errors of these meters.

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D) systems with the mutual correction.

In connection with the use of integrated systems appears the question about the mutual correction of radio engineering and self-contained meters with which measured value y_p of radio engineering system or its derivative are introduced into the self-contained meter where they are utilized as corrective command.

Self-contained meter most frequently has the limited aperture, beyond limits of which it either is saturated or it gives exaggerated errors. Mutual aggregation makes it possible to throttle/taper the required range of the linearity of self-contained meter and from this point of view its use is expedient.

In [46] is given an example of the mutual correction utilized in practice of the Doppler meter of the ground speed of aircraft and inertial calculator of path. As a result of mutual correction are created the most favorable working conditions of both meters. Because of the use of corrective commands of radio engineering system frequently it is very simple to attain the stable operation of inertial meters [16]), which by other methods to carry out very complicatedly or even is impossible.

Let us find the transfer functions of signals and errors in the

system with the mutual correction under the assumption about the fact that both systems are linear. The radio engineering servo system consists of components/links W_1 and W_2; into it is introduced signal y_a from the output of the self-contained system through the matching filter $H_1(D)$ (Fig. 2.15).

Self-contained meter is the servo system W_3, W_4, where from the servo radio engineering system through the matching filter H_2 is introduced signal y_p (Fig. 2.15).

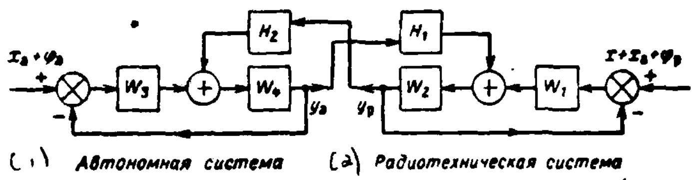

Fig. 2.15.

Key: (1). Self-contained system. (2). Radio engineering system.

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The system in question relates to the class of systems with the incomplete information, since the input of radio engineering system enters the supplementary signal x. Certainly, it is possible the principle of mutual correction of using, also, for the systems with the perfect information.

Proceeding from the diagram in Fig. 2.15, we obtain:

$$y_a = \frac{1}{1 + W_a}[W_a x_a + W_a \varphi_a + H_2 W_4 y_p], \qquad (2.89)$$

$$y_p = \frac{1}{1 + W_p}[W_1 x + W_p \varphi_a + H_1 W_2 y_a], \qquad (2.90)$$

where

$$W_a = W_3 W_4, \qquad (2.91)$$

$$W_p = W_1 W_2. \qquad (2.92)$$

solving these equations relatively y_a and y_p, we will obtain

$$y_a = \frac{[(1+W_p)W_a + H_2W_4W_p]x_a + (1+W_p)W_a\varphi_a +}{(1+W_a)(1+W_p) -}$$

$$\frac{+H_2W_4W_p(\varphi_p + x)}{-H_1H_2W_2W_4}, \qquad (2.93)$$

$$y_p = \frac{W_p(1+W_a)(x + x_a + \varphi_p) + H_1W_2W_a x_a + W_a H_1 W_2 \varphi_a}{(1+W_a)(1+W_p) - H_1H_2W_2W_4}. \qquad (2.94)$$

The conditions of invariance let us select so that in the signals y_a and y_p value x_a, general/common/total for both meters, would pass to the output without the distortion, i.e., the error of reproduction did not contain this value. They lead to the following equalities:

$$H_1W_2 = H_2W_4 = 1. \qquad (2.95)$$

Taking into account these equalities from relationships/ratios (2.93) and (2.94), we will obtain

$$y_a = x_a + (1-\Phi_a)x + \Phi_a\varphi_a + (1-\Phi_a)\varphi_p, \qquad (2.96)$$

$$y_p = x_a + \Phi_p x + (1-\Phi_p)\varphi_a + \Phi_p\varphi_p, \qquad (2.97)$$

where it is marked:

$$\Phi_a = \frac{(1+W_p)W_a}{W_a + W_p + W_aW_p}, \qquad (2.98)$$

$$\Phi_p = \frac{(1+W_a)W_p}{W_a + W_p + W_aW_p}. \qquad (2.99)$$

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Let us record finally expressions for the errors of the reproduction: for it is self-contained the system

$$z_{авт} = x_a - y_a = -(1-\Phi_a)x - (1-\Phi_a)\varphi_a -$$

$$-\Phi_a\varphi_p = -\frac{1}{1+W'_э}x + \frac{1}{1+W'_э}\varphi_a + \frac{W'_э}{1+W'_э}\varphi_p; \qquad (2.100)$$

for the radio engineering system:

$$z_{\text{раз}} = x + x_a - y_p = (1-\Phi_p)x - \Phi_p\varphi_p - (1-\Phi_p)\varphi_a = \\ = \frac{1}{1+W''_э}x - \frac{W''_э}{1+W''_э}\varphi_p - \frac{1}{1+W''_э}\varphi_a, \quad (2.101)$$

where

$$W'_э = \frac{W_a}{W_p}(1+W_p), \quad W''_э = \frac{W_p}{W_a}(1+W_a). \quad (2.102)$$

According to its structure formula (2.101) completely corresponds to formula (2.54) for the invariant integrated system with the incomplete information and one connection/communication.

Thus, under the conditions indicated we obtain the same expressions for the errors, as in the mentioned case. Expressions for the components of the errors in formula (2.101) coincide with expressions (2.56), (2.57) and (2.58), during replacement of W on $W''_э$.

With complexing it is necessary to provide stability and the necessary quality of control in both systems. Without stopping on these questions, let us record expression for the characteristic equation of the locked system. If

$$W_a = \frac{P_a(D)}{Q_a(D)}; \quad W_p = \frac{P_p(D)}{Q_p(D)}, \quad (2.103)$$

where P_a, Q_a, P_p, Q_p — polynomials relative to the symbol of differentiation D, then characteristic equation will take the form

$$[P_a(D)Q_p(D)P_p(D)Q_a(D)+P_a(D)P_p(D)]_{D=\lambda}=0.$$

From this equation it is located condition, with which the system remains stable with the mutual correction.

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Chapter 3.

ANALYSIS OF INTEGRATED SYSTEMS.

In this chapter are examined the errors of integrated systems, caused by errors in two meters with different statistical characteristics of their errors. Are rated/estimated possible gains and are located the conditions of their realization.

Are examined two cases. The first of them concerns the combined processing of the signals of two independent linear meters. The second relates to the case, when one of the meters is radio engineering, moreover are taken into consideration all essential specific special features/peculiarities of this meter with the random character of input signals, including chance of transmission factor.

§ 3.1.

TRANSFER FUNCTIONS OF FILTERS DURING THE COMBINED PROCESSING OF THE SIGNALS OF SEVERAL METERS.

Let there be n meters of one and the same value x. Each of them possesses the specific errors (instrument/tool, systematic). Besides

this on the input of sensor operate outside interferences. Therefore as a result of measurements output signal $x_i (i=1, 2, \ldots, n)$ of each meter besides value x(t) contains error z_i, so that

$$x_i(t)=x(t)+z_i(t). \qquad (3.1)$$

These signals enter the filters with transfer functions $\Phi_i(D)$, and then they are summarized (Fig. 3.1).

It is of interest to determine possible during this treatment to raise the accuracy of the measurements of value x and what conditions they must satisfy in this case the transfer functions of filters.

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To the diagram in Fig. 3.1 are reduced integrated automatic systems with the perfect information.

For the evaluation/estimate of accuracy are utilized different criteria. With the errors $z_i(t)$, which can be described by the stationary functions of time, most frequently is utilized the criterion of the minimum of variance of error. Expression for the output signal takes the form

$$y(t)=\sum_{i=1}^{n} \Phi_i(D)\, x_i(t). \qquad (3.2)$$

We will consider the errors of meters $z_i(t)$ the stationary

random functions of time with the zero average/mean values. The parameters of filters $\Phi_i(D)$ must be selected so that the output signal would contain the undistorted input value x(t), and interference component of z(t) of output signal had minimum value, i.e.,

$$y(t)=x(t)+z(t), \tag{3.3}$$

moreover

$$<z^2(t)>=<[y(t)-x(t)]^2>=\min. \tag{3.4}$$

This system of filters it is possible to call optimum.

Filters are chosen so that the input signal is transmitted without the dynamic errors. Furthermore during the assignment of the characteristics of filters it is necessary each time to have in mind satisfaction of the conditions of physical feasibility. Let us record output signal in the form

$$y(t)=\sum_{i=1}^{n}\Phi_i(D)[x(t)+z_i(t)], \tag{3.5}$$

or

$$y(t)=\sum_{i=1}^{n}\Phi_i(D)x(t)+\sum_{i=1}^{n}\Phi_i(D)z_i(t). \tag{3.6}$$

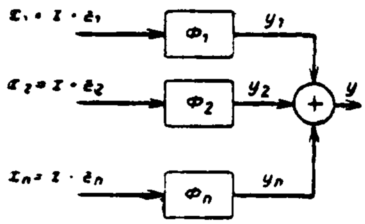

Fig. 3.1.

Error must not contain the components, which depend on x(t). This means that must be satisfied the condition

$$\sum_{i=1}^{n} \Phi_i(D) = 1, \qquad (3.7)$$

and then the error of the system

$$z(t) = \sum_{i=1}^{n} \Phi_i(D) z_i(t). \qquad (3.8)$$

Condition (3.7) sets substantial limitations on the selection of filters $\Phi_i(D)$.

Let us consider these limitations [42]. We will consider that the transfer function of each filter is the relation of two polynomials:

$$\Phi_i(p) = \frac{A_i(p)}{B_i(p)} = \frac{A_i(p)}{\prod_{j=1}^{N_i}(p-\alpha_{ij})}, \qquad (3.9)$$

where the degree of the polynomial of numerator $A_i(p)$ does not exceed the degree of the polynomial of denominator $B_i(p)$. Transfer function (3.9) is recorded in the form of the Laplace transform by replacing the differential operator D to the parameter of transformation p.

Taking into account (3.7), let us record:

$$\frac{A_1(p)}{\prod_{j=1}^{N_1}(p-\alpha_{1j})}+\frac{A_2(p)}{\prod_{j=1}^{N_2}(p-\alpha_{2j})}+\ldots+\frac{A_n(p)}{\prod_{j=1}^{N_n}(p-\alpha_{nj})}. \quad (3.10)$$

Hence we find:

$$A_1(p)\prod_{\substack{j=1\\k=2,3,\ldots,n}}^{N_k}(p-\alpha_{kj})+A_2(p)\prod_{\substack{j=1\\k=1,3,4,\ldots,n}}^{N_k}(p-\alpha_{kj})+$$

$$+\ldots+A_n(p)\prod_{\substack{j=1\\k=1,2,3,\ldots,n-2,n-1}}^{N_k}(p-\alpha_{kj})=$$

$$=\prod_{\substack{j=1\\k=1,2,\ldots,n}}^{N_k}(p-\alpha_{kj}). \quad (3.11)$$

or in the more compact recording

$$\sum_{i=1}^{n}A_i(p)\prod_{\substack{j=1\\k\neq i}}^{N_k}(p-\alpha_{kj})=\prod_{\substack{j=1\\k=1,2\ldots,n}}^{N_k}(p-\alpha_{kj}). \quad (3.12)$$

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Let us make in (3.11) value of p equal to the j pole of the 1st filter: $p=\alpha_{1j}$. Then all components/terms/addends on the left side (3.11), except the first, and the right side of this equality become zero, i.e.,

$$A_1(\alpha_{1j})\prod_{j=1}^{N_k}(p-\alpha_{kj})=0,\quad k=2,3,\ldots,N_n. \qquad (3.13)$$

Since $A_1(\alpha_{1j})\neq 0$ (otherwise numerator and denominator of transfer function Φ_1 would contain identical factors and it was possible to shorten them), we obtain

$$\left[\prod_{j=1}^{N_2}(p-\alpha_{2j})\prod_{j=1}^{N_3}(p-\alpha_{3j})\ldots\prod_{j=1}^{N_k}(p-\alpha_{nj})\right]_{p=\alpha_{1j}}=0. \quad (3.14)$$

This is possible only in such a case, when among roots α_{2j}, α_{3j} ... is located at least one, equal to α_{1j}. Thus, among the poles of the transfer functions of the 2nd, the 3rd, ..., the n-th of filters will be located at least one, which coincides with one of the poles α_{1j} of the 1st filter. Discussing analogously relative to each filter, we come to the conclusion the fact that the poles of any filter $\Phi_i(p)$ compulsorily are contained among the poles of remaining filters [42]; the addition of any new (n+1) filter does not increase the assortment of the poles, which already contain in the system from the n filters; bands (n+1) filter compulsorily will be located among poles n of filters.

Hence it follows that the denominators of the transfer functions of filters with the highest degree (with exception of the filters, which contain multiple ones poles) coincide. This means that the filters of the highest orders (order of filter is called the degree

of the polynomial of denominator) they can be distinguished only by the polynomials of numerators (with exception of filters with the multiple roots).

Since the degree of the polynomial of numerator in real filters cannot be higher than degree of the polynomial of denominator, at least in one of the filters of higher order (switching on filters with the multiple poles) the degrees of the polynomials of numerator and denominator coincide. Only in this case can be performed equality (3.7).

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If all filters have the identical order N, then in the absence of multiple poles, the denominators of the transfer functions of all filters (3.9) are identical, i.e.,

$$\Phi_i(p)=\frac{A_i(p)}{\prod\limits_{j=1}^{N}(p-\alpha_j)}, \qquad (3.15)$$

where the denominator on i depends.

In this case from (3.7) we find:

$$\frac{\sum\limits_{i=1}^{n}A_i(p)}{\prod\limits_{j=1}^{N}(p-\alpha_j)}=\frac{\sum\limits_{i=1}^{n}A_i(p)}{b_Np^N+b_{N-1}p^{N-1}+\ldots+b_1p+b_0}=1 \qquad (3.16)$$

and

$$\sum_{i=1}^{n} A_i(p) = b_N p^N + b_{N-1} p^{N-1} + \ldots + b_0.$$

The degree of the polynomial of numerator must be equal to N, i.e.,

$$\sum_{i=1}^{n} A_i(p) = a_N p^N + a_{N-1} p^{N-1} + \ldots + a_0.$$

whence follows

$$a_0 = b_0, \quad a_1 = b_1, \ldots \quad a_N = b_N, \qquad (3.17)$$

moreover

$$a_j \neq 0, \; j = 1, 2, \ldots, N.$$

Here $a_0, a_1, \ldots, a_n$ – sum of the coefficients, confronting appropriate by degrees p of the numerators of transfer functions of all filters.

This rule is suitable for checking the coefficients of the polynomial of the numerators of transfer functions of filters.

Thus, if transfer function of one of two filters

$$\Phi_1(p) = \frac{c_2 p^2 + c_0}{(Tp+1)(\tau p+1)},$$

and of the other

$$\Phi_2(p) = \frac{c_2}{(Tp+1)(\tau p+1)},$$

but another then such filters cannot satisfy condition (3.7), since in the polynomials of numerator there is no term with first degree of p [is not satisfied condition (3.17), i.e., $a_1=0$].

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If numerator $\Phi_i(p)$ will contain term c_1p, then such filters can satisfy conditions (3.17), for example, filters with the transfer functions

$$\Phi_1(p) = \frac{T\tau p^2 + (T+\tau)p + (1-c_2)}{T\tau p^2 + (T+\tau)p + 1},$$

$$\Phi_2(p) = \frac{c_2}{T\tau p^2 + (T+\tau)p + 1}.$$

It is easy to ascertain that conditions (3.17) here are satisfied and

$$\Phi_1(p) + \Phi_2(p) = 1.$$

The given example corresponds to the diagram, examined above (page 66).

The obtained conditions sometimes are conveniently presented in another form [42]. Assuming that in the filters the multiple poles are absent, let us record the transfer functions of filters in the form:

$$\Phi_i(p) = \alpha_{i0} + \sum_{k=1}^{N_i} \frac{A_i(p_k)}{B'_i p_k (p - p_k)} = \alpha_0 + \sum_{k=1}^{N} \frac{\alpha_{ik}}{p - p_k}. \quad (3.18)$$

Let N number of poles of filter Φ_n of higher order $(N_i < N)$. Since all of the pole of filters Φ_i are contained among the poles of filter Φ_N. ; us supplement each of sums (3.18) with members of the type $\alpha_{ir}/p - p_r$, where r - being missing pole, and $\alpha_{ir} = 0$. Then instead of (3.18) we record:

$$\Phi_i(p) = \alpha_{i0} + \sum_{k=1}^{N} \frac{\alpha_{ik}}{p - p_k} = \sum_{k=0}^{N} \frac{\alpha_{ik}}{p - p_k}, \quad (3.19)$$

where conditionally it is counted $p-p_0=1$.

Utilizing, further, condition (3.7), we will obtain:

$$\sum_{i=1}^{n} \Phi_i(p) = \sum_{i=1}^{n} a_{i0} + \sum_{i=1}^{n} \sum_{k=1}^{N} \frac{a_{ik}}{p-p_k} = 1. \tag{3.20}$$

Changing, further, the order of addition in (3.20), let us find

$$\sum_{i=1}^{n} a_{i0} + \sum_{k=1}^{N} \frac{1}{p-p_k} \sum_{i=1}^{n} a_{ik} = \sum_{i=1}^{n} a_{i0} + $$
$$+ \frac{1}{p-p_1} \sum_{i=1}^{n} a_{i1} + \ldots + \frac{1}{p-p_N} \sum_{i=1}^{n} a_{iN} = 1. \tag{3.21}$$

equality (3.21) can be performed, if only

$$\sum_{i=1}^{n} a_{i0} = 1, \quad \sum_{i=1}^{n} a_{ik} = 0, \quad k = 1, 2, \ldots, N. \tag{3.22}$$

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We come to the conclusion that the sums of all coefficients of the expansion of the transfer functions of filters into partial fractions in the identical poles of all filters must be equal to zero.

Conditions (3.22) present another form of the recording of those obtained of earlier the conditions, which are the consequence of requirement (3.7). By representation (3.20) and conditions (3.22) are conveniently used during the theoretical studies, since notation of the transfer functions of filters proves to be symmetrical.

Let us pass to the examination by interference component of output signal and will record expression for the dispersion of output value σ_y^2.

It is obvious,

$$\sigma_y^2 = \langle z^2(t) \rangle. \tag{3.23}$$

Substituting in (3.23) the expression for error (3.8) let us

write:

$$\sigma_y^2 = < \left[\sum_{i=1}^{n} \Phi_i(D) z_i(t)\right]^2 > =$$
$$= < \sum_{i=1}^{n} \sum_{j=1}^{n} \Phi_i(D) z_i(t) \Phi_j(D) z_j(t) >. \qquad (3.24)$$

Changing the order of addition and operation of mathematical expectation and taking into account that operators Φ_i and Φ_j are linear, let us record expression (3.24) in the form:

$$\sigma_y^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} < \Phi_{it'}(D) \Phi_{jt''}(D) z_i(t) z_j(t) > =$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \Phi_{it'}(D) \Phi_{jt''}(D) < z_i(t') z_j(t'') >. \qquad (3.25)$$

In the latter/last expression each of the operations Φ_i and Φ_j is conducted respectively on argument t' and t", moreover after these operations are produced, it is necessary to assume t'=t"=t.

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The order of the taking of operations is indifferent, for example, first can be realized operation $\Phi_{it''}$ (in this case t' it is examined the fixed/recorded parameter), and then obtained result it undergoes operation $\Phi_{it'}$ (moreover t" it must be considered as the fixed/recorded parameter), or, on the contrary, first is conducted operation $\Phi_{it'}$, and then $\Phi_{it''}$. The expression

$$<[z_i(t')z_j(t'')]>=<z'_iz''_j>$$

is a correlation function of two signals

$$z_i(t')=z'_i \overset{(1)}{\text{и}} z_j(t'')=z''_j;$$

$$<z'_i\cdot z''_j>=R_{ij}(t',t'')=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}z'_iz''_j\cdot f_2(z'_i;z''_j,t',t'')dz'_idz''_j, \qquad (3.26)$$

Key: (1). and.

where f_2 - two-dimensional probability density for random functions $z_i(t)$, $z_j(t)$.

In the absence of the cross correlation between errors z_i in the separate channels (precisely this case is examined subsequently) expression (3.26) takes the form:

$$<[z'_iz''_i]>=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}z'_iz''_if_2(z',z'',t',t'',)dz'dz''=$$

$$=R_{ii}(t',t''), \qquad (3.27)$$

since all integrals with $i\neq j$ prove to be equal to zero. Thus, with of the statistically independent, the errors separate meters

$$\sigma_y^2=\sum_{i=1}^{n}\Phi_{it'}(D)\Phi_{it''}(D)R_{ii}(t',t''), \qquad (3.28)$$

where after the realization of operations $\Phi_{it'}$, $\Phi_{it''}$, it is necessary to assume t'=t" =t. If random functions $z_i(t)$ are stationary, R_{ii} will

be the function only of a difference in arguments t'-t"=τ. The computation of dispersion (3.28) produces usually some computational difficulties; for their facilitation it is convenient to resort to the apparatus of the two-dimensional Laplace transform, in detail studied and substantiated in book [41]. It is further necessary to minimize dispersion σ_y^2 with satisfaction of conditions (3.22) or (3.7).

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Let us consider the simplest case when as the filters are utilized proportional inertia-free elements/cells with the constant transmission factors K_i. Then from (3.28) we obtain

$$\sigma_y^2=\sum_{i=1}^{n} K_iK_iR_{ii}(0)=\sum_{i=1}^{n} K_i^2\sigma_i^2, \qquad (3.29)$$

where σ_i^2 - variance of error of input signals.

In this simplest case for the determination of minimum (3.29) under the condition

$$\sum_{i=1}^{n} K_i=1 \qquad (3.30)$$

it is possible either with the help of (3.30) to exclude "excess" variable/alternating and to seek unconditional minimum σ_y^2 or to use the method of Lagrange's indefinite factors, composing the function

$$F=\sum_{i=1}^{n}\left\{K_i^2\sigma_i^2+\lambda\left[\sum_{i=1}^{n}K_i-1\right]\right\}.$$

Equating to zero partial derivatives $\partial F/\partial K_i$ and utilizing conditions (3.30), we find optimum values K_i:

$$K_1=\frac{1}{1+\frac{\sigma_1^2}{\sigma_2^2}+\ldots+\frac{\sigma_1^2}{\sigma_n^2}},$$

$$K_2=\frac{1}{1+\frac{\sigma_2^2}{\sigma_1^2}+\frac{\sigma_2^2}{\sigma_3^2}+\ldots+\frac{\sigma_2^2}{\sigma_n^2}}\cdots$$

or

$$K_i=\frac{1}{\sigma_i^2\sum_{i=1}^{n}\frac{1}{\sigma_i^2}} \qquad (3.31)$$

and minimum value of the dispersion

$$\sigma^2_{y\,\text{um}}=\frac{1}{\sum_{i=1}^{n}\frac{1}{\sigma_i^2}}. \qquad (3.32)$$

Weight coefficient K_i conversely is proportional to dispersion σ_i^2 of the measuring error of each sensor. This conclusion/output is obvious physically: the less accurately sensor realizes a measurement of value x, the less must be its "specific contribution" to the general/common/total signal.

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If meter-sensors have errors with the identical dispersions σ^2, then $K_i = K = \frac{1}{n}$, while $\sigma^2_{y_{\text{НЗМ}}} = \sigma^2/n$. i.e. we come to the well known result: variance of error decreases in n, and rms value $\sqrt{n}$ times.

The method of using the information of several sources usually examined is called simple averaging.

§3.2. Error analysis in the system with two meters.

Let us consider an important special case of the use of those obtained earlier relationships/ratios for the error analysis of two meters whose signals are treated using the method of filtration (Fig. 3.1), moreover each of the filters $\Phi_1(D)$ and $\Phi_2(D)$ we will consider single-section, i.e.

$$\Phi_1(D) = \frac{1}{TD+1}, \quad \Phi_2(D) = 1 - \Phi(D) = \frac{TD}{TD+1}.$$

Let us assume that the correlation functions of the errors of the first and second meter are described by the expressions

$$R_{11}(t', t'') = R_1(\tau) = \sigma_1^2 e^{-\alpha_1 |\tau|}, \qquad (3.33)$$

$$R_{22}(t', t'') = R_2(\tau) = \sigma_2^2 e^{-\alpha_2 |\tau|}. \qquad (3.33')$$

Formula (3.28) in this case takes the form:

$$\sigma_y^2 = \Phi_{1t'}(D)\,\Phi_{1t''}(D)\,R_{11}(t'\,t'') + \\ + \Phi_{2t'}(D)\,\Phi_{2t''}(D)\,R_{22}(t't''), \qquad (3.34)$$

where it is necessary to assume after conducting of the corresponding operations in Φ $t'=t''=t$, and in R $t'-t''=\tau$.

The first member of sum (3.34) is the dispersion of the result of occurrence of the error of the first meter [with the correlation function (3.33) through filter $\Phi_1(D)$, and the second - error of the second meter through filter $\Phi_2(D)$].

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In accordance with (3.34) these dispersions are written/recorded as follows:

$$\sigma_{y1}^2 = \int_0^{t'}\int_0^{t''} g_1(t'-u)\,g_1(t''-v)\,R_{11}(u, v)\cdot du\,dv =$$

$$= \sigma_1^2 \int_0^t\int_0^t g_1(t-u)\,g_1(t-v)\,e^{-\alpha_1|u-v|}\,du\,dv, \qquad (3.35)$$

$$\sigma_{y2}^2 = \sigma_2^2 \int_0^t\int_0^t g_2(t-u)\,g_2(t-v)\,e^{-\alpha_2|u-v|}\,du\,dv, \qquad (3.36)$$

where g_1 and g_2 - pulse transient responses of the first and second filters.

Virtually, however, for the computation it is simpler to use the known expressions

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$$\left.\begin{aligned}\sigma_{y1}^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |\Phi_1(j\omega)|^2 S_1(\omega)\,d\omega,\\ \sigma_{y2}^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |\Phi_2(j\omega)|^2 S_2(\omega)\,d\omega\end{aligned}\right\} \tag{3.37}$$

and

$$\sigma_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2, \tag{3.38}$$

where $S_1(\omega)$ and $S_2(\omega)$ - the spectral densities of the errors of the first and second meter which for correlation functions (3.33) and (3.33') are expressed as follows:

$$S_1(\omega) = \frac{2\alpha_1\sigma_1^2}{\alpha_1^2+\omega^2}, \tag{3.39}$$

$$S_2(\omega) = \frac{2\alpha_2\sigma_2^2}{\alpha_2^2+\omega^2}. \tag{3.40}$$

After satisfying integration with the help of the known relationships/ratios (see [3] or [43]) from (3.37)-(3.40), we will obtain

$$\sigma_y^2 = \frac{\sigma_1^2}{\alpha_1 T+1} + \frac{\sigma_2^2\alpha_2 T}{\alpha_2 T+1}. \tag{3.41}$$

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Let us find the optimum value of the time constant $T=T_{o1}$ of accumulated error ensuring minimum value $\sigma^2_{y\,min}$ of dispersion (3.41). By differentiating σ_y^2 on T and by equalizing derivative zero, we will

obtain

$$T=T_0=\frac{1-s}{s\alpha_2-\alpha_1}=\frac{1}{\sqrt{\alpha_1\alpha_2}}\cdot\frac{\sqrt{\sigma_2^2\alpha_2}-\sqrt{\sigma_1^2\alpha_1}}{\sqrt{\sigma_1^2\alpha_2}-\sqrt{\sigma_2^2\alpha_1}}, \qquad (3.42)$$

$$\sigma^2_{y\,мин}=\frac{\sigma_1^2(s\alpha_2-\alpha_1)+\sigma_2^2 s\alpha_2(1-s)}{s(\alpha_2-\alpha_1)}, \qquad (3.43)$$

where

$$s=\sqrt{\frac{\sigma_1^2\alpha_1}{\sigma_2^2\alpha_2}}. \qquad (3.44)$$

Let us introduce designations for the relation of dispersions and values, which characterize rate of fall of the correlation functions:

$$r=\frac{\sigma_1^2}{\sigma_2^2},\quad d=\frac{\alpha_1}{\alpha_2}. \qquad (3.45)$$

Then expressions (3.42) and (3.43) are recorded as follows:

$$T_0=\frac{1-\sqrt{rd}}{\alpha_2(\sqrt{rd}-d)}, \qquad (3.46)$$

$$\sigma^2_{y\,мин}=\sigma_2^2\,\frac{1+r-2\sqrt{rd}}{1-d}=\sigma_1^2\,\frac{1+r-2\sqrt{rd}}{r(1-d)}. \qquad (3.47)$$

Before analyzing the obtained relationships/ratios, let us consider case of d=1, i.e., $\alpha_1=\alpha_2=\alpha$, when correlation functions are characterized by only dispersions.

In this case σ_y^2 does not have minimum ($T_0<0$) and σ_y^2 monotonically increases (with r<1) or decreases (with r>1) from σ_1^2 to σ_2^2 (Fig. 3.2).

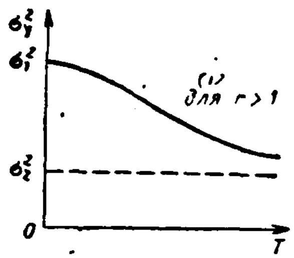

Fig. 3.2.

Key: (1). for.

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With $T \rightarrow 0$ the band of filter Φ_1 is expanded, approaching infinity; therefore all noises of the 1st meter without difficulty are passed to the output, while the noises of the 2nd meter - completely cut themselves and accumulated error $\sigma_y^2 = \sigma_1^2$, as this follows from (3.41).

With $T \rightarrow \infty$, on the contrary, completely are suppressed the noises of the 1st sensor, but are passed the noises of the 2nd and $\sigma_y^2 = \sigma_2^2$.

With identical correlation functions when r=1 d=1, $\sigma_y^2 = \sigma_1^2 = \sigma_2^2$ independent of value T.

Let us pass to the examination of the maximum gain which can be obtained during the complex use of two filters of the I order (case d=1 in this case is eliminated).

From (3.46) it follows that T_0 will be positive with fulfilling of following inequalities:

$$\text{— при}^{(1)}\ rd > 1, \qquad r < d, \qquad (3.48)$$
$$\text{— при}^{(1)}\ rd < 1, \qquad r > d. \qquad (3.49)$$

Key: (1). with.

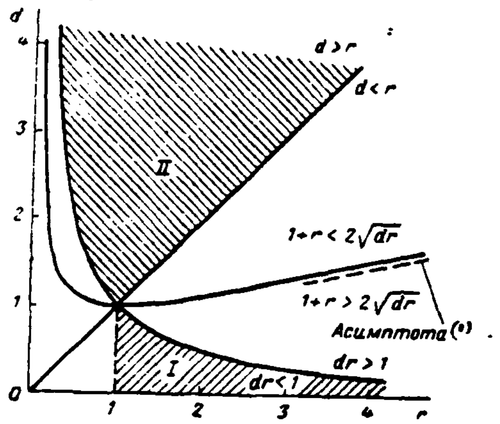

Fig. 3.3.

Key: (1). Asymptote.

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From (3.47) it follows that $\sigma^2_{\text{умин}}$ will be positive, if

$$-\text{при}^{(1)}\ d<1 \qquad 1+r>2\sqrt{rd}, \qquad (3.50)$$
$$-\text{при}^{(1)}\ d>1 \qquad 1+r<2rd. \qquad (3.51)$$

Key: (1). with.

Fig. 3.3 depicts regions I and II of values r and d, corresponding to actual conditions of reaching minimum σ^2_y. Hence it is apparent that the boundaries of these

regions are lines rd=1 and r=d; whereas conditions (3.50) and (3.51) real limitations do not set.

With r<1 σ_v^2 it can reach the minimum only for d>1/r>1.

If r>1, then there are two regions of values d, where σ_v^2 reaches the minimum; one of them corresponds to values of d<1, another d>1.

For the characteristic of the efficiency of filtration let us introduce coefficient $\gamma_{эф}$ [42]:

$$\gamma_{эф}=\frac{\sigma_{y0}^2}{\sigma_{y\,мин}^2}, \qquad (3.52)$$

equal to the ratio of dispersion σ_{y0}^2 without the filters (with the averaging) to dispersion $\sigma_{y\,мин}^2$ of system with the use of filters.

In accordance with (3.32) with n=2

$$\sigma_{y0}^2=\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}. \qquad (3.53)$$

Substituting in expression (3.52) of value for σ_{y0}^2 and $\sigma_{y\,мин}^2$, we will obtain:

$$\gamma_{эф}=\frac{r(d-1)}{(r+1)[(2\sqrt{rd}-(r+1)]}. \qquad (3.54)$$

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It is clear that the domain of existence $\gamma_{эф}$ it will be the same as and for $\sigma^2_{y\,мин}$ (I and II in Fig. 3.3).

The forseeable results can be obtained, on the basis of the examination of the graph/diagrams of dependence $\gamma_{эф}$ (r) at different values of d (Fig. 3.4).

If d is small (d<1), then $\gamma_{эф} < 0{,}5$, moreover value r, at which minimum σ^2_{y} exists, they lie/rest within narrow limits and with decrease of d decrease. The use of filters leads to the loss. To this it is easy to give simple physical explanation.

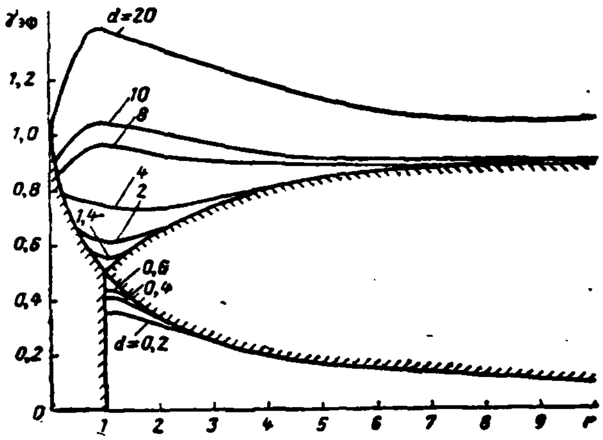

Рис. 3.4.

Fig. 3.4.

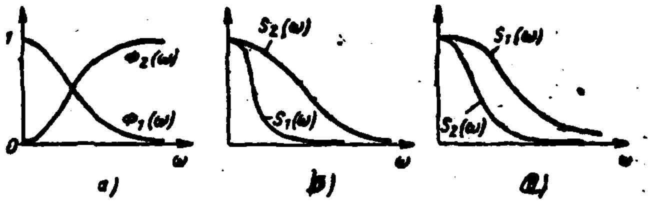

Fig. 3.5.

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With d<1 (Fig. 3.5b), i.e., when the time of correlation $\tau_{\kappa 1}=1/\alpha_1$ is more than the time of correlation $\tau_{\kappa 2}=1/\alpha^2_2$, the interference, which enters filter Φ_1, will be more than "narrow-band", than the interference, which enters filter Φ_2. This situation will be unfavorable, since the more broadband interference enters filter Φ_2 of higher frequencies, and narrower-band - to filter Φ_1 of lower

frequencies, and gain cannot be obtained here (Fig. 3.5).

On the contrary, with d>1 (Fig. 3.5c) situation will be favorable, since the broadband interference enters filter Φ_1 of lower frequencies, and narrow-band - to filter Φ_2 of higher frequencies, and filters will exert the effective filtering action and here it is possible to expect the specific gain (Fig. 3.5). Actually/really, this gain $(\gamma_{э\phi}>1)$ occurs with d>1, the value of gain increasing with increase in d. As it follows from Fig. 3.4, maximum occurs at values of r, close to 1.

Assuming/setting in expression (3.54) r=1 for large d, we will obtain

$$\gamma_{э\phi}=0,25\sqrt{d}.$$

Thus, for obtaining the essential gain necessary that d would be is sufficiently great, i.e., so that the spectra essentially would differ by width.

The intermediate range of values d of large ones, but close to unity, is not of essential interest, since and here gain it is not obtained.

One should, however, note that with r, close to 1, $\gamma_{э\phi}$ has not a

maximum, but a minimum.

Let us consider further the case when the input of filter enters very broadband signal, within the limit - white noise with an intensity of $S_1=(\omega)=N$ (let us note that dimensionality N does not coincide with the dimensionality σ^2_2:$[N]=[\sigma^2_2]\cdot[s]$). Direct transition/junction to the limit during recording S_1 in the form (3.39) is impossible.

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Therefore let us represent the value of the dispersion of output signal as the sum

$$\sigma^2_y = \sigma^2_{y1} + \sigma^2_{y2} = \frac{N}{2T} + \frac{\omega^2_2 \alpha_2 T}{\omega^2_2 T + 1}, \qquad (3.55)$$

where first term is obtained by computing the integral

$$\sigma^2_{y1} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{N d\omega}{|j\omega T + 1|^2}.$$

First term of this sum from increase of T falls, the second - monotonically increases. Therefore can exist certain value $T=T_0$, at which total dispersion σ^2_y reaches its minimum value. Differentiating (3.55) on T and equalizing derivative zero, it is possible to find the optimum value of the constant of the filter

$$T_0 = \frac{\sqrt{N}\,\sigma_2\sqrt{2\alpha_2} - N\alpha_2}{\alpha_2\,(N\alpha_2 - \sigma_2^2)} \qquad (3.56)$$

and the minimum value of the dispersion:

$$\sigma^2_{\nu\,\text{мин}} = \frac{N\alpha_2\,(N\alpha_2 - \sigma_2^2)\,(\sigma_2\sqrt{2N\alpha_2} - \sigma_2^2) + 2\sigma_2^2\,(\sigma_2\sqrt{2N\alpha_2} - N\alpha_2)^2}{2\,(\sigma_2\sqrt{2N\alpha_2} - \sigma_2^2)\,(\sigma_2\sqrt{2N\alpha_2} - N\alpha_2)}. \qquad (3.57)$$

Optimum value exists only, when $T_0>0$. Then from (3.56)

$$\frac{N\alpha_2}{2} < \sigma_2^2 < N\alpha_2$$

or

$$0{,}5 < \frac{\sigma_2^2}{N\alpha_2} < 1. \qquad (3.58)$$

S3.3. Synthesis of integrated system with two meters.

Together with the problem of analysis examined of practical interest is the problem of synthesis which can be formulated thus.

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There are two meters of one and the same value $x(t)$ with errors $z_1(t)$ and $z_2(t)$, being statistically independent stationary random functions of time.

It is necessary to synthesize filters Φ_1 and Φ_2 with the constant parameters, which are subordinated to condition (3.7), for which the dispersion of accumulated error will be minimum. The solution of this problem will make it possible to establish/install, to what extent are distant the filters examined from the optimum ones, that ensure the best (in the sense of the minimum of the root-mean-square error) filtration.

Let us record expression for the output error. In accordance with expressions (3.35) and (3.36) with t'=t" =t

$$\sigma_v^2 = \int_0^t\int_0^t g_1(t-u)\, g_1(t-v)\, R_1(u-v)\, du\, dv + \\ + \int_0^t\int_0^t g_2(t-u)\, g_2(t-v)\, R_2(u-v)\, du\, dv = \sigma_{v1}^2 + \sigma_{v2}^2. \tag{3.59}$$

It is here taken into consideration, that $R_{11}=R_1$ and $R_{22}=R_2$ - function only of a difference in arguments u-v.

Since is correct the equality

$$\int_0^{t'}\int_0^{t''} g(t'-\xi)\, g(t''-\tau)\, R(\tau-\xi)\, d\tau\, d\xi = \\ = \int_0^{t'}\int_0^{t''} g(u)\, g(v)\, R(u-v)\, du\, dv, \tag{3.60}$$

of what easy to be convinced by the replacement of the variable/alternating $t'-\xi=u$ and $t''-\tau=v$, then (3.59) it is possible to

rewrite as follows:

$$\sigma_y^2(t)=\int_0^t\int_0^t g_1(u)\,g_1(v)\,R_1(u-v)\,du\,dv+$$

$$+\int_0^t\int_0^t g_2(u)\,g_2(v)\,R_2(u-v)\,du\,dv. \qquad (3.61)$$

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For the determination of stationary error it is necessary to pass to the limit of this expression with $t\to\infty$.

In expression (3.61) the pulse transient responses $g_1(u)$ and $g_2(u)$ are not independent variables, since $\Phi_1(D)$ and $\Phi_2(D)$ are connected with conditions (3.7).

Since

$$g_1(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_1(j\omega)\,e^{j\omega t}\,d\omega,$$

then

$$g_2(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_2(j\omega)\,e^{j\omega t}\,d\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}[1-\Phi_1(j\omega)]\,e^{j\omega t}\,d\omega=\delta(t)-$$

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_1(j\omega)\,e^{j\omega t}\,d\omega=\delta(t)-g_1(t). \qquad (3.62)$$

Thus, the pulse transient responses of filters are connected with relationship/ratio (3.62).

Replacing in (3.61) function g_2 by its expression (3.62), for second integral (3.61) we will obtain

$$\sigma^2_{v2}=\int_0^t [\delta(v)-g_1(v)]\,dv\int_0^t [\delta(u)-$$

$$-g_1(u)]\,R_s(u-v)\,du=R_s(0)-2\int_0^t g_1(u)\,R_s(u)\,du+$$

$$+\int_0^t\int_0^t g_1(u)\,g_1(v)\,R_s(u-v)\,du\,dv. \qquad (3.63)$$

In the transformations conducted we used the following property of δ-function:

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\,dx=f(x_0). \qquad (3.64)$$

and also is changed the order of integration for u and v.

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Taking into account (3.64) the dispersion of accumulated error will be recorded as follows:

$$\sigma^2_v=R_s(0)-2\int_{-\infty}^{\infty} g_1(u)\,R_s(u)\,du+$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(u)\,g_1(v)\,R(u-v)\,du\,dv, \qquad (3.65)$$

where

$$R(u-v)=R_1(u-v)+R_2(u-v)$$

- total correlation function of the errors of meters.

In expression (3.65) upper limits t are replaced to the infinite ones, since with the synthesis it is proposed to seek the minimum of the steady error, and lower limits are replaced on - ∞, since with $t<0$, $g_1(t)=g_2(t)=0$.

For determination of weight function $g_{10}(t)$, which minimizes variance of error σ_v^2, introduces certain arbitrary function $g_\lambda(t)$ with the help of the relationship/ratio:

$$g_1(t)=g_{10}(t)+\lambda g_\lambda(t),$$

where λ - parameter, different from zero, and $\lambda g_\lambda(t)$ - variation $g_r(t)$.

Substituting for $g_1(t)$ its expression in (3.65), we find conditions, with which value σ_v^2 (expressed now through $g_0(t)$ and λ) for λ≠0 has a minimum. For this derivative $d\sigma_v^2/d\lambda$ is equated zero.

As a result of the transformations (see for example, [43], page 149 or [3], page 260) we come to the known equation of Wiener-Kolmogorov for the unknown function g_{10}:

$$g_{1*}=\begin{cases}\int\limits_{-\infty}^{\infty} g_{1*}(v)\,R(t-v)\,dv-R_1(t) & \overset{(1)}{\text{при}}\ t\geqslant 0,\\ 0 & \overset{(1)}{\text{при}}\ t<0,\end{cases}$$

key: (1). with.

which gives the minimum value of error.

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The problem of synthesis in the setting indicated repeatedly was solved in the literature (see, for example [43, 3, 18] and the solution of integral equation is known sufficiently good.

Most convenient in the practical sense is solution in the form of the transfer function of optimum filter.

For the integral equation in question this function will be recorded

$$\Phi_1(p)=\frac{1}{\Psi(p)}\left[\frac{S_2\left(\frac{p}{j}\right)}{\Psi(-p)}\right]_+. \qquad (3.66)$$

Here $\Phi_1(p)$ - unknown transfer function of the filter, which has optimum weight function $g_{1*}(t)$; $S_2(p/j)=S_2(\omega)$ - the spectral density of interference at the input of the 2nd filter, which corresponds to the correlation function $R_2(\tau)$ (for the integral equation of

Wiener-Kolmogorov - sum of the spectral density of signal and mutual spectral density of signal and interference at the input):

$$\Psi(p)\Psi(-p)=|\Psi(j\omega)|^2=S_1(\omega)+S_2(\omega) \qquad (3.67)$$

- the total spectral density of interferences at the input of filters, corresponding to the total correlation function $R(\tau)=R_1(\tau)+R_2(\tau)$ (for the integral equation of Wiener-Kolmogorov - sum of the spectral densities of signal and interference).

Formula $[S_2(p/j)/\Psi(-p)]_+$ means that in the expression, included in the brackets, are held only those components/terms/addends of function $\Psi(p)$, poles of which lie/rest at the left half-plane. Poles and zero functions $\Psi(p)$ also lie/rest at the left half-plane.

Let us note that sum $S_1(\omega)+S_2(\omega)$ can be represented in the form of product $\Psi(p)\Psi(-p)$ only with satisfaction of the specified condition ~~(p2Ii~~ Peli -Wiener)which usually is performed for the majority of the in practice utilized approximations of spectral densities.

Let us consider two cases:

1. The spectral densities of interferences $z_1(t)$ and $z_2(t)$ are described by expressions (3.33) and (3.33').

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Since $p=j\omega$, then $S_1(\omega)$ and $S_2(\omega)$ will be recorded in the form:

$$S_1\left(\frac{p}{j}\right)=\frac{2\alpha_1\sigma_1^2}{\alpha_1^2-p^2};$$
$$S_2\left(\frac{p}{j}\right)=\frac{2\alpha_2\sigma_2^2}{\alpha_2^2-p^2}. \qquad (3.68)$$

Let us find further $\Psi(p)\ \Psi(-p)$, for which let us represent sum of S_1+S_2 as

$$\Psi(p)\Psi(-p)=S_1\left(\frac{p}{j}\right)+S_2\left(\frac{p}{j}\right)=$$
$$=\frac{\sqrt{2}\,(A+Bp)}{(\alpha_1+p)(\alpha_2+p)}\cdot\frac{\sqrt{2}\,(A-Bp)}{(\alpha_1-p)(\alpha_2-p)},$$

where
$$\left.\begin{aligned} A^2&=\alpha_1\alpha_2(\sigma_1^2\alpha_2+\sigma_2^2\alpha_1),\\ B^2&=\alpha_1\sigma_1^2+\alpha_2\sigma_2^2.\end{aligned}\right\} \qquad (3.69)$$

hence it follows that

$$\Psi(p)=\frac{\sqrt{2}(A+Bp)}{(\alpha_1+p)(\alpha_2+p)};$$
$$\Psi(-p)=\frac{\sqrt{2}(A-Bp)}{(\alpha_1-p)(\alpha_2-p)}.$$

Let us record further relation $S_2(\omega)/\Psi(-p)$, after expanding it on the sum of components, one of which $[S_2(\omega)/\Psi(-p)]_+$ has pole only in the left half-plane complex variable p and the other $[S_2(\omega)/\Psi(-p)]_-$ - only in the right.

$$\frac{S_2\left(\frac{p}{j}\right)}{\Psi(-p)}=\frac{2\alpha_2\sigma_2^2(\alpha_1-p)}{\sqrt{2}(\alpha_1+p)(A-Bp)}=[\ldots]_+ + [\ldots]_- =$$
$$=\sqrt{2}\alpha_2\sigma_2^2\left\{\left[\frac{M}{\alpha_1+p}+\frac{N}{A-Bp}\right]\right\}.$$

Values M and N are easily located by the usual reception/procedure:

$$M=\frac{\alpha_1+\alpha_2}{A+\alpha_2 B};\ N=\frac{B(\alpha_1+\alpha_2)}{A+\alpha_2 B}-1. \qquad (3.70)$$

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We are interested only in the component

$$\left[\frac{S_2\left(\frac{p}{j}\right)}{\Psi(-p)}\right]_+=\sqrt{2}\alpha_2\sigma_2^2\frac{M}{\alpha_1+p},$$

having the only pole $p=-\alpha_1$ in the left half-plane.

Now optimum transfer function will be recorded easily:

$$\Phi_1(p)=\frac{\left[\frac{S_2\left(\frac{p}{j}\right)}{\Psi(-p)}\right]_+}{\Psi(p)}=\frac{\alpha_2\sigma_2^2(\alpha_1+p)M}{A+Bp},$$

or

$$\Phi_1(D)=\frac{K_1(\tau'_1 D+1)}{\tau''_1 D+1}, \qquad (3.71)$$

where

$$K_1=\frac{\alpha_1\alpha_2\sigma_2^2 M}{A};\ \tau'_1=\frac{1}{\alpha_1};$$
$$\tau''_1=\frac{B}{A}=\frac{\sqrt{d(1+dr)}}{\alpha_1\sqrt{(d+r)}}.$$

Further immediately we find

$$\Phi_2(p) = 1 - \Phi_1(p) = \frac{(A - \alpha_1\alpha_2\sigma_2^2 M) + p(B - \alpha_2\sigma_2^2 M)}{A + Bp},$$

or

$$\Phi_2(D) = \frac{(\tau''_1 - K_1\tau'_1)D + (1 - K_1)}{\tau''_1 D + 1}. \qquad (3.72)$$

Optimum filters are the boosting/forcing components/links. Therefore at them it cannot be arrived via the appropriate selection of the parameters of filters $\Phi_1(j\omega)$ and $\Phi_2(j\omega)$, examined above.

Comparison of the error of synthesized system with the minimum error, calculated [1] earlier [formula (3.43)], in the general case it is difficult.

FOOTNOTE [1]. Expression for $\sigma^2_{\varphi\,\text{мин}}$ is obtained below (page 118). ENDFOOTNOTE.

Therefore let us compare the appropriate results for a particular example. Let us assume $\sigma_1=\sigma_2=\sigma$, and $\alpha_1/\alpha_2=9$. Then for synthesized system $\sigma^2_{\varphi\,\text{ош}} = 0{,}3\sigma^2$, and for the unsynthesized system minimum error $\sigma^2_{\varphi\,\text{мин}} = 0{,}5\sigma^2$.

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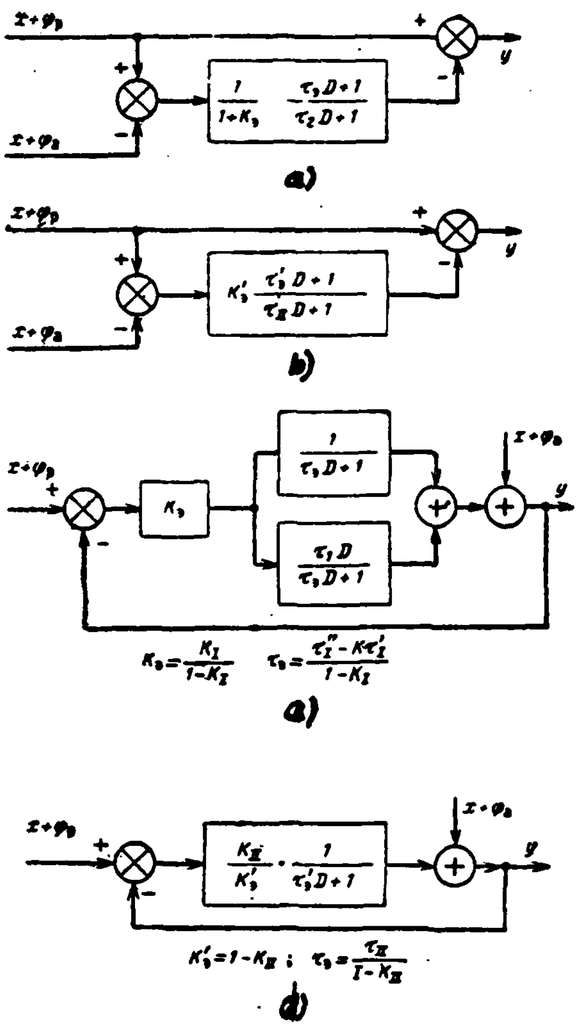

Fig. 3.6.

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Findings make it possible to construct the block diagrams of optimum system on three described diagrams. Assuming/setting errcੰs z_1 and z_2 by respectively equal ones to φ_p and φ_a, for the diagram of filtration let us record

$$y = x + \Phi_1\varphi_p + \Phi_2\varphi_a.$$

Comparing this equality with expressions (2.13), (2.32) and (2.4), we will obtain:

- for the diagram of the compensation

$$\Phi_1 = 1 - F;\ \Phi_2 = F;$$

- for the diagram of the aggregation

$$\Phi_1 = \Phi_p = \frac{W}{1+W};\ \Phi_2 = \frac{1}{1+W}.$$

In accordance with these formulas taking into account (3.71) and (3.72) are constructed the block diagrams of compensation (Fig. 3.6a) and aggregation (3.6b).

2. Error $z_1(t)$ is very broadband and can be approximated by white noise; error $z_2(t)$ it is as before approximated by expression (3.40), i.e.

$$S_1(\omega) = N;\ S_2(\omega) = \frac{2\alpha_2\sigma_2^2}{\alpha_2^2 + \omega^2}.$$

Then the analogous method of actions leads to the following expressions:

$$\Psi(p)=\frac{L+\sqrt{N}p}{\alpha_2+p};\ \Psi(-p)=\frac{L-\sqrt{N}p}{\alpha_2-p},$$

where

$$L=\sqrt{\alpha_2^2(N\alpha_2+2\sigma_2^2)},$$

$$\frac{S_3(p/j)}{\Psi(-p)}=2\alpha_2\sigma_2^2\left[\frac{M}{\alpha_2+p}+\frac{Q}{L-\sqrt{N}p}\right]=[\ldots]_{+}+[\ldots]_{-}.$$

In the latter/last expression

$$M=\frac{1}{L+\alpha_2\sqrt{N}},\quad Q=\frac{\sqrt{N}}{L+\alpha_2\sqrt{N}},$$

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From then

$$\Phi_1(p)=\frac{2\alpha_2\sigma_2^2M}{L+\sqrt{N}p}=\frac{2\alpha_2\sigma_2^2\tau_{11}}{N(1+\alpha_2\tau_{11})}\cdot\frac{1}{(\tau_{11}p+1)},$$

or

$$\Phi_1(D)=\frac{K_{11}}{\tau_{11}D+1}. \qquad (3.73)$$

Here

$$T=\frac{\sqrt{N}}{L}=\frac{\sqrt{N}}{\sqrt{\alpha_2(2\sigma_2^2+N\alpha_2)}};$$

$$K_{11}=1-\alpha_2\tau_{11}=\frac{2\alpha_2\sigma_2^2M}{L}.$$

Further let us find

$$\Phi_2(p) = 1 - \Phi_1(p),$$

or

$$\Phi_2(D) = \frac{\tau_{11}D + (1 + K_{11})}{\tau_{11}D + 1}. \qquad (3.74)$$

Hence it follows that the structure of filters Φ_1 and Φ_2, examined above, for this case was optimum.

In this case, however, the transmission factor of filter with p=0 must be different from unity. Thus, the synthesized system of filters differs from that examined earlier (page 107) by the fact that the transmission factor of the filter was selected previously equal to unity, that is must lead to greater accumulated error in comparison with the synthesized filters, which have transfer functions (3.73) and (3.74).

The corresponding diagrams of filtration and the aggregations, constructed with the comparison of equalities (3.73) and (3.74) with expressions (2.13)obtained previously, (2.32) and (2.4), make it

possible to construct the optimum diagrams of compensation (Fig. 3.6c) and aggregation (3.6d).

It is not difficult to compute smallest possible error for the synthesized system.

Variance of error

$$\sigma^2_{\text{уопт}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} [|\Phi_1(j\omega)|^2 S_1(\omega) + |\Phi_2(j\omega)|^2 S_2(\omega)] d\omega.$$

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For case 1 let us record

$$\sigma^2_{y\,\text{out}} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K_I^2\,|\tau_I' j\omega + 1|^2\, 2\alpha_1\sigma_1^2}{|\tau_I'' j\omega + 1|^2(\alpha_1^2+\omega^2)}\,d\omega +$$

$$+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\alpha_2\sigma_2^2\,|(\tau_I'' - K_I\tau_I')\,j\omega + (1-K_I)|^2}{|\tau_I'' j\omega + 1|^2(\alpha_2^2+\omega^2)}\,d\omega =$$

$$= 2\alpha_1\sigma_1^2 K_1^2\,\frac{\alpha_1\tau_I' + \tau_{II}}{2\,\tau_I'\alpha_1(1+\alpha_1\tau_I'')} +$$

$$+ 2\alpha_2\sigma_2^2(1-K_I)^2\,\frac{\alpha_2(\tau_I'' - \tau_I' K_I)^2 + \tau_I'')}{2\alpha_2\tau_I''(1+\alpha_2\tau_I'')}. \qquad (3.75)$$

For case 2 we will have:

$$\sigma^2_{y\,\text{out}} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K_{II}^2\sqrt{N}}{|\tau_{II} j\omega + 1|^2}\,d\omega +$$

$$+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|j\omega\tau_{II} + (1+K_{II})|^2\, 2\alpha_2\sigma_2^2}{|\tau_{II} j\omega + 1|^2(\alpha_2^2+\omega^2)}\,d\omega =$$

$$= \frac{K_{II}^2\sqrt{N}}{2\tau_{II}} + 2\alpha_2\sigma_2^2\,\frac{\tau_{II}\alpha_2 + (1+K_{II})^2}{2\alpha_2(1+\tau_{II}\alpha_2)}.$$

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At the assigned values of N, α_2, σ_2 this value of variance of error is smallest possible in the class of stationary linear filters.

If we, however, remove/take limitations on the constancy of the parameters of filters, then position about the maximum gain can significantly be changed.

In [20, page 153] is examined the case of the optimization of the integrated system in which they are included:

a) the inertial meter, which gives second derivative D^2x_a and which possesses the noise, concentrated in the zero frequency (correlation function - constant value);

b) the radio engineering meter of position x_a with the broadband (white) noise (correlation function - δ-function).

As output value of system serves speed Dx_a.

Is solved the problem about the synthesis of system with the

variable parameters, speed Dx_a intended for the evaluation/estimate with the minimum dispersion at a constant value of acceleration $\ddot{x}_a$.

Error with the obtained structure asymptotically decreases, vanishing, while in the optimum system with the constant parameters it it remains constant. However, the solution of problem relates to a special case of motion with the uniform acceleration.

It should be noted that the utilized in [20] method of synthesis leads to the need of solving the integral equations, which can be fulfilled analytically only in the isolated special cases. Most frequently for the solution of such problems it is necessary to resort to the use of mathematical machines (digital or simulating) or to run the appropriate experiments.

Problems examined above related to the class of the systems with the perfect information about the measured value, in which for the identification of the parameters of the matching filters it suffices it was sufficient to consider only the statistical characteristics of the errors of meters φ_a and φ_p.

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In the systems with the incomplete information it is necessary

to consider that besides these two comprising vital importance has the dynamic error

$$z_д = \frac{1}{1 + W} x,$$

caused by the passage of the component x through the locked radio engineering system. Therefore in the optimum system must be provided the minimum of the accumulated error. Usually problem to the optimum is solved for the most probable or most difficult conditions for the work of system.

In this case signal x is considered as the regular or assigned random function, and is located optimum of pole, which gives the minimum of the sum of the variances of error $\langle z_a^2 \rangle$ and $\langle z_p^2 \rangle$ and square of dynamic error $\langle z_д^2 \rangle$.

If the parameters of system or the parameters of interferences are changed during the motion of object, then for guaranteeing the minimum it is necessary to change the parameters of the system of self-contained meters, and also matching filters (system with the self-adjusting).

The examination of these cases exceeds the scope of this book.

§3.4. Analysis of integrated systems with the random transmission

factor.

The measuring elements/cells of the radio engineering devices/equipment, which form part of integrated systems, are nonlinear and have the randomly changing in the time transmission factor. The chance of the parameter (transmission factor) is caused, first of all, by the fluctuations of the signals echo from the target and by the action of interferences on radio channel. The analysis of integrated systems without taking into account the fluctuations of the parameters gives the overstated characteristics of their freedom from interference.

The problem of the investigation of automatic systems with the random parameters consists of the determination of the statistical characteristics of the solutions of differential equations with the coefficients, which are the random functions of time.

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During the theoretical analysis of similar systems appear fundamental difficulties, since at present there are no general mathematical methods of obtaining the exact solutions of linear differential equations with the random coefficients, with exception of first-order equations. Therefore considerable place during the

analysis of such systems occupy different approximation methods [44], based on what changes in the parameters are assumed/set either by small ones or slow ones.

During the investigation of linear automatic control systems first-order with the random parameters possible the exact solution of the corresponding differential equation. The analysis of the named parametric systems for the case when the fluctuating parameter has the arbitrary law of probability distribution it is carried out by G. Khellgren [28], and in work [35] analysis is propagated to the case of the linearly changing input signal.

As it was already shown above, the block diagram of integrated, linear system with astaticism of the 1st order takes the form, represented in Fig. 3.7, where for the certainty of the reasonings is depicted the case of correction by the signal of speed.

In contrast to those examined earlier the case, here $K_{\text{вр}}(t)$ is the random function of time.

On one input of integrated system operate the signals:

$x_{\text{а}}(t)$ - signal information about which can be obtained also from the self-contained meter; x(t) - signal information about which

cannot be obtained from the self-contained meter;

$f_p(t)$ - signal, caused by interferences on the radio channel and converted to the output of measuring element/cell. About the representation of measuring element/cell by this equivalent it was mentioned into §1.1.

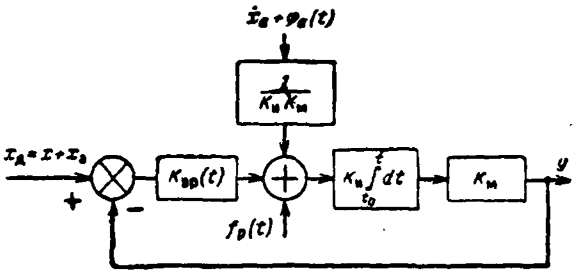

Fig. 3.7.

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The detailed substantiation of this for the case, when measuring element/cell is temporary/time discriminator, is given in Appendix.

On another input of integrated system they operate the signal: $\dot{x}_a(t)$. - signal of the self-contained meter, which is the first time derivative of signal $x_a(t)$; $\varphi_a(t)$ the signal, caused by the errors of self-contained meter. Value y(t) is the output signal of the integrated system of automatic radio equipment.

The differential equation, which corresponds to the block diagram, depicted in Fig. 3.7 takes the form

$$\frac{dy(t)}{dt} + K_v(t)\,y(t) = \frac{dx_a(t)}{dt} + K_v(t)\,[x(t) + x_a(t)] + Kf_p(t) + \varphi_a(t). \quad (3.76)$$

where $K_v(t)=K_{ap}(t)\cdot K$ - transmission factor of system; $K=K_n K_M$ - transmission factor of the determined part of the system.

Thus, steering signal input for the integrated system in question is signal $x_n(t)=x(t)+x_a(t)$; the designation/purpose of system itself is the reproduction (measurement) of it with the smallest errors

$$z(t)=x_n(t)-y(t).$$

Since the system is investigated in the linear conditions, to it let us use the principle of superposition in accordance with which the output signal of system y(t) can be found as the sum of the partial output signals:

$$y(t)=y_1(t)+y_2(t)+y_3(t)+y_4(t), \qquad (3.77)$$

where partial output signals $y_1(t)$, $y_2(t)$, $y_3(t)$ are caused by action individually of input signals $x(t)$, $f_p(t)$, $\varphi_a(t)$ respectively, and $y_4(t)$ is caused by action in the set of signal $x_a(t)$ on one input of integrated system and signal

$$\dot{x}_a(t)=\frac{dx_a(t)}{dt}$$

on its another input. As it was shown above, as a result of satisfaction of the condition of invariance, signal $x_a(t)$ at the output of system is reproduced without the distortions, i.e.

$$y_1(t) = x_1(t).$$

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Let us find signal $y_1(t)$, in this case we assume that on the input of system operates only signal x(t).

Then equation (3.76) takes the form

$$\frac{dy_1}{dt} + K_v(t)\,y_1(t) = K_v(t)\,x(t) \qquad (3.78)$$

and is the linear differential equation of the 1st order.

Process $y_1(t)$ at the output of the system, described by this equation, can be found as the roll of weight function and input signal (for example, see [3]). For the system under the zero initial conditions process $y_1(t)$ is written/recorded as

$$y_1(t) = \int_0^t g_1(t;\ \tau)\,x(\tau)\,d\tau, \qquad (3.79)$$

where $g_1(t;\ \tau)$ - the weight (pulse transient) function of system, which is the function of two variable/alternating: the moment/torque of the application/appendix of input pulse τ and moment/torque of observing the process at the output of system t.

Let us note that for the parametric system weight function depends on t and τ, but it is not the function of a difference in these arguments as in the case of linear systems with the constant parameters.

Weight function $g_1(t; \tau)$ can be defined as the solution of equation (3.78) with zero initial conditions and $x(t)\delta(t)$ (here $\delta(t)$ it is the delta function:

$$g_1(t; \tau) = K_v(\tau)\, e^{-K\int_{\tau}^{t} K_{\text{вр}}(\rho)\, d\rho} \qquad (3.80)$$

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In perfect analogy are determined components output signal $y_2(t)$ and $y_3(t)$, caused respectively by interferences on the radio channel and errors of the self-contained meter:

$$y_2(t) = \int_0^t g_2(t; \tau) f_p(\tau)\, d\tau, \qquad (3.81)$$

$$y_3(t) = \int_0^t g_3(t, \tau) \varphi_a(\tau)\, d\tau, \qquad (3.82)$$

where

$$g_2(t; \tau) = K e^{-K\int_{\tau}^{t} K_{\text{вр}}(\rho)\, d\rho} \qquad (3.83)$$

- weight function of system, which corresponds to input signal $f_p(t)$;

$$g_3(t, \tau) = e^{-K\int_{\tau}^{t} K_{\text{вр}}(\rho)\, d\rho} \qquad (3.84)$$

- weight function of system for the signal, determined $\varphi_a(t)$.

It is here everywhere assumed that the system begins to work in moment/torque $t_0=0$.

For further analysis it is useful to note that in accordance with expressions (3.80), (3.83) and (3.84) we have:

$$g_1(t,\ \tau)=\frac{\partial}{\partial\tau}g_3(t;\ \tau), \qquad (3.85)$$

$$g_2(t,\ \tau)=Kg_3(t,\ \tau). \qquad (3.86)$$

Further problem consists in the determination of the statistical characteristics of the output signal y(t), namely - its mathematical expectation and dispersion. Since the components $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$ are not depended, then computations let us lead individually for each component.

Most simply are determined the statistical characteristics of the component of the output signal $y_3(t)$, which is caused by the errors of self-contained meter.

On the basis (3.82) and taking into account that $K_{sp}(\dot{t})$ and $\varphi_a(t)$ are not depended, we have

$$\langle y_3(t)\rangle=\int_0^t\langle g_3(t;\ \tau)\rangle\langle\varphi_a(\tau)\rangle\, d\tau. \qquad (3.87)$$

In is examined/considered case $\langle\varphi_a(\tau)\rangle=0$, then $\langle y_a(t)\rangle=0$.

Average from the square of output value is represented as [2]:

$$\langle[y_a(t)]^2\rangle=\int_0^t\int_0^t\langle g_a(t,\ \tau)\,g_a(t,\ \nu)\rangle\langle\varphi_a(\tau)\,\varphi_a(\nu)\rangle\,d\tau d\nu, \quad (3.88)$$

where $\langle g_a(t,\ \tau)\,g_a(t,\ \nu)\rangle$ - two-dimensional initial moment/torque of the second order of weight function $g_a(t;\ \tau)$, of that corresponding to zero shift/shear of argument t; $\langle\varphi_a(\tau)\,\varphi_a(\nu)\rangle=R_\varphi(\tau-\nu)$ - two-dimensional initial moment/torque of the second order of input effect $\varphi_a(t)$.

Since $\langle y_a(t)\rangle=0$, the taking into account (3.88) expression for the dispersion of the component $y_a(t)$ can be recorded in the form:

$$\sigma_3^2(t)=\int_0^t\int_0^t\langle g_a(t,\ t-\tau)\,g_a(t,\ t-\nu)\rangle\,R_\varphi(\tau-\nu)\,d\tau d\nu. \quad (3.89)$$

In the steady-state mode/conditions

$$\sigma_{3y}^2=\int_0^\infty\int_0^\infty\langle g_a(t;\ t-\tau)\,g_a(t;\ t-\nu)\rangle\,R_\varphi(\tau-\nu)\,d\tau d\nu.$$

Since $\varphi_a(t)$ - mixing effect, $\sigma_3^2(t)$ is the variance of error of the measurement of controlling input effect $x_\text{и}(t)=x(t)+x_a(t)$, of those caused by the errors of self-contained meter.

During the determination of the statistical characteristics of the measuring errors of input effect $x_1(t)$, of those caused by uncompensated component x(t), the latter we represent in the form

$$x(t)=x_0+vt+\beta(t), \qquad (3.90)$$

where x_0 and v - constant numbers, respectively equal to the initial value of value x(t) and the speed of its change with time;

β(t) - the random function with the zero average/mean value, which is determining random changes in value x(t).

In our examination for simplicity let us assume that β(t) - normal stationary function.

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Taking into account (3.79) and (3.90) it is possible to write

$$y_1(t)=y_{11}(t)+y_{12}(t)+y_{13}(t), \qquad (3.91)$$

where

$$y_{11}(t)=\int_0^t g_1(t,\ \tau)\,x_0 d\tau, \qquad (3.92)$$

$$y_{12}(t)=\int_0^t g_1(t,\ \tau)\,v\tau d\tau, \qquad (3.93)$$

$$y_{13}(t)=\int_0^t g_1(t,\ \tau)\,\beta(\tau) d\tau. \qquad (3.94)$$

For the average/mean value of the component $y_{11}(t)$ according to (3.85) we have

$$\langle y_{11}(t)\rangle = x_0 \int_0^t \left[\frac{\partial}{\partial \tau}\langle g_2(t, \tau)\rangle\right] d\tau = x_0 \langle g_2(t, t)\rangle - x_0 \langle g_2(t, 0)\rangle.$$

From examination (3.84) it follows that

$$\langle g_2(t, t)\rangle = 1,$$

then

$$\langle y_{11}(t)\rangle = x_0 [1 - \langle g_2(t, 0)\rangle]. \quad (3.95)$$

By force (3.88) the average from the square of the component $y_{11}(t)$ is equal to:

$$\langle y_{11}^2\rangle = \int_0^t \int_0^t \langle g_1(t, \tau) g_1(t, \nu)\rangle x_0^2 \, d\tau d\nu.$$

On the basis (3.85) for the average/mean value from the square of weight function $g_1(t, \tau)$ we will obtain

$$\langle g_1(t, \tau) g_1(t, \nu)\rangle = \frac{\partial^2}{\partial \tau \partial \nu} \langle g_2(t, \tau) g_2(t, \nu)\rangle. \quad (3.96)$$

Then

$$\langle y_{11}^2(t)\rangle = x_0^2 \int_0^t \int_0^t \left[\frac{\partial^2}{\partial \tau \partial \nu} \langle g_2(t, \tau) g_2(t, \nu)\rangle\right] d\tau d\nu = x_0^2 \langle [1 - g_2(t, 0)]^2\rangle.$$

Taking into account the obtained expression the dispersion of

the component $y_{11}(t)$ will take the form:

$$\sigma_{11}^2(t) = \langle y_{11}^2(t)\rangle - [\langle y_{11}(t)\rangle]^2 = \\ = x_0^2 \langle g_2^2(t, 0)\rangle - x_0^2 [\langle g_2(t, 0)\rangle]^2. \quad (3.97)$$

In steady-state mode/conditions $\langle g_2^2(t, 0)\rangle = 0$. Therefore dispersion $\sigma_{11y}^2 = 0$.

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The component y_{12} is caused by the input effect vt, where v=const:

$$\langle y_{12}(t)\rangle = v\int_0^t \langle g_1(t, \tau)\rangle \tau d\tau = \\ = vt - v\int_0^t \langle g_2(t, \tau)\rangle d\tau, \quad (3.98)$$

$$\langle y_{12}^2(t)\rangle = \int_0^t\int_0^t \langle g_1(t, \tau) g_1(t, \nu)\rangle v^2 \tau\nu d\tau d\nu = \\ = v^2 \int_0^t\int_0^t \left[\frac{\partial^2}{\partial\tau\partial\nu} \langle g_2(t, \tau) g_2(t, \nu)\rangle\right] \tau\nu d\tau d\nu = \\ = v^2 t^2 - 2v^2 t \int_0^t \langle g_2(t, \tau)\rangle d\tau + \\ + v^2 \int_0^t\int_0^t \langle g_2(t, \tau) g_2(t, \nu)\rangle d\tau d\nu. \quad (3.99)$$

Taking into account (3.98) and (3.99) for the dispersion of the component $y_{12}(t)$, it is possible to record:

$$\sigma_{12}^2(t) = v^2 \int_0^t\int_0^t \langle g_2(t, \tau) g_2(t, \nu)\rangle d\tau d\nu - \\ - v^2 \left[\int_0^t \langle g_2(t, \tau)\rangle d\tau\right]^2. \quad (3.100)$$

In the steady-state mode/conditions

$$\sigma^2_{12y}=\sigma^2\int_0^\infty\int_0^\infty \langle g_2(t,\ \tau)g_2(t,\ \nu)\rangle\, d\tau d\nu -$$
$$-\sigma^2\left[\int_0^t \langle g_2(t,\ \tau)\rangle\, d\tau\right]^2. \qquad (3.101)$$

The component of the output signal $y_{13}(t)$ is caused by the function $\beta(t)$, which reflects random changes in the input signal. Since $\langle\beta(t)\rangle=0$,

$$\langle y_{13}(t)\rangle=0 \qquad (3.102)$$

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The measuring error of signal $x_{\text{п}}(t)$, caused by component $\beta(t)$ is defined as

$$z_{13}=\beta(t)-y_{13}(t),$$

then

$$\sigma^2_{13}(t)=<z^2_{13}(t)>-[<z_{13}(t)>]^2=R_\beta(0)-$$
$$-2\int_0^t<g_1(t,t-\tau)>R_\beta(\tau)d\tau+$$
$$+\int_0^t\int_0^t<g_1(t,t-\tau)g_1(t,t-\nu)>R_\beta(\tau-\nu)d\tau d\nu. \qquad (3.103)$$

In the steady-state mode/conditions

$$\sigma^2_{13y}=R_\beta(0)-2\int_0^\infty<g_1(t,t-\tau)>R_\beta(\tau)d\tau+$$
$$+\int_0^\infty\int_0^\infty<g_1(t,t-\tau)g_1(t,t-\nu)>R_\beta(\tau-\nu)d\tau d\nu.$$

Let us determine the statistical characteristics the components $y_2(t)$, caused by interferences along the radio engineering channel. By *analogy, for example,* with $y_1(t)$ it is possible to obtain

$$< y_2(t) > = 0, \text{ так как}^{(1)} < f_p(t) > = 0, \quad (3.104)$$

$$\sigma_2^2(t) = < y_2^2(t) > = \int_0^t \int_0^t < g_2(t, t-\tau) g_2(t, t - \nu) > R_f(\tau - \nu) d\tau d\nu,$$

Key: (1). since.

where in accordance with (3.86)

$$< g_2(t, t-\tau) g_2(t, t-\nu) > = K^2 < g_3(t, t-\tau) g_3(t, t-\nu) >.$$

In the steady-state mode/conditions

$$\sigma_{2y}^2 = \int_0^\infty \int_0^\infty < g_2(t, t-\tau) g_2(t, t-\nu) > R_f(\tau - \nu) d\tau d\nu. \quad (3.105)$$

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Since $f_p(t)$ - mixing effect, then $\sigma_2^2(t)$ is the variance of error of the measurement of signal x(t), caused by interferences on the radio channel. Knowing the statistical characteristics of the components of output signal or measuring errors of controlling, input signal $x_n(t)$, it is possible to obtain the statistical characteristics of the complete measuring errors of signal $x_n(t)$.

Taking into account the obtained average/mean values of components, the average/mean value of the output signal y(t) is equal to:

$$\langle y(t)\rangle = x_{\text{д}}(t) + x_0[1 - \langle g_{з}(t,0)\rangle] + + vt - v\int_0^t \langle g_{з}(t,\tau)\rangle d\tau.$$

Then the average/mean value of the measuring error of input signal $x_{\text{д}}(t)$ can be recorded as

$$\langle z(t)\rangle = \langle x_{\text{д}}(t)\rangle - \langle y(t)\rangle = = x_0\langle g_{з}(t,0)\rangle + v\int_0^t \langle g_{з}(t,\tau)\rangle d\tau. \quad (3.106)$$

In the steady-state mode/conditions

$$\langle z_{y}\rangle = v\int_0^t \langle g_{з}(t,\tau)\rangle d\tau. \quad (3.107)$$

On the basis of the obtained formulas for the variances of error of the measurement of input signal $x_{\text{д}}(t)$ from separate components it is possible to record expression for the general/common/total variance of error of the measurement of signal $x_{\text{д}}(t)$:

$$\sigma_{\varepsilon}^2(t) = \sigma_{11}^2(t) + \sigma_{12}^2(t) + \sigma_{13}^2(t) + \sigma_2^2(t) + \sigma_3^2(t). \quad (3.108)$$

In steady-state mode/conditions the expression of the variance of error of ranging takes the form

$$\sigma_{\varepsilon y}^2 = \sigma_{12y}^2 + \sigma_{13y}^2 + \sigma_{2y}^2 + \sigma_{3y}^2. \quad (3.109)$$

Expressions (3.107) and (3.109) show that in the steady-state mode/conditions the average/mean value and the variance of error of the measurement of signal $x_{д}(t)$ from the time do not depend, if $K_{вр}(t)$ - stationary function of time.

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From formula (3.108) it is evident that in the expression for the variance of error as a result of fluctuations of transmission factor $K_{вр}(t)$ appeared two supplementary components which in the formula for the analogous dispersion, which characterizes system with the constant parameters, are absent (see Chapter 5).

Most essential is the component of dispersion $\sigma_{12}^{2}(t)$, which is different from zero and in the steady-state mode/conditions.

Using the methodology, presented in [28], let us determine the statistical characteristics of weight function of parametric system with astaticism of the 1st order in the case when $K_{вр}(t)$ - normal stationary process.

The legitimacy of the assumption indicated is substantiated in the Application/Appendix.

In accordance with (3.84) it is possible to write

$$<g_{з}(t,\tau)> = <\exp\left\{-K\int_{\tau}^{t} K_{\text{вр}}(\rho)\,d\rho\right\}>, \tag{3.110}$$

$$<g_{з}(t,\tau)\,g_{з}(t,\nu)> = <\exp\left\{-K\int_{\tau}^{t} K_{\text{вр}}(\rho)\,d\rho - K\int_{\nu}^{t} K_{\text{вр}}(\rho)\,d\rho\right\}>. \tag{3.111}$$

Random factor of amplification $K_{\text{вр}}(t)$ can be represented in the form of the sum of its average/mean value and stationary normal narrow-band noise with the arbitrary spectrum and the zero mathematical expectation. With such assumptions, as shown by S. O. Rice [45], factor of amplification $K_{\text{вр}}(t)$ is equal to

$$K_{\text{вр}}(t) = K_{\text{вр}0} + \sum_{n=1}^{N}[a_n\cos n\,\Delta\omega t + b_n\sin n\,\Delta\omega t], \tag{3.112}$$

where $K_{\text{вр}0} = <K_{\text{вр}}(t)>$; $K_{\text{вр}0}>0$ from the physical considerations; a_n and b_n - components of the assemblies of independent normally distributed random variables with the zero mathematical expectation.

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The statistical characteristics of coefficients a_n and b_n can be recorded as

$$<a_n> = <b_n> = 0;\ <a_n^2> = <b_n^2> = G(n\,\Delta\omega)\,\Delta\omega;$$
$$<a_n a_m> = <b_n b_m> = 0,\ \text{если}\ m \neq n;$$
$$<a_n b_m> = 0\ \text{для всех}\ m\ \text{и}\ n.\ (2)$$

Key: (1). if. (2). for all m and n.

Value $G(n\Delta\omega)$ is the average/mean power, obtained by the averaging of components $K_{sp}(t)$, in the range of frequencies $n\Delta\omega<\omega<(n+1)\Delta\omega$.

After substituting into expressions (3.110) (3.111) the resolution of factor of amplification $K_{sp}(t)$ in the form (3.112) and after completing after computations passage to the limit when $N\to\infty$ and $\Delta\omega\to d\omega$, we will obtain final formulas for the statistical characteristics of weight function $g_s(t,\tau)$:

$$<g_s(t,t-\tau)>=\exp\left\{-K_0\tau+\frac{1}{2}\frac{K_0^2}{K_{sp0}^2}\tau^2 a(\tau)\right\}, \tag{3.113}$$

$$<g_s(t,t-\tau)g_s(t,t-\nu)>=\exp\left\{-K_0(\tau+\nu)+\right.$$
$$\left.+\frac{K_0}{K_{sp0}^2}\left[\tau^2 a(\tau)+\nu^2 a(\nu)-\frac{1}{2}(\tau-\nu)^2 a(\tau-\nu)\right]\right\}, \tag{3.114}$$

where

$$a(\tau)=\int_0^\infty \frac{4\sin^2\frac{\omega\tau}{2}}{\omega^2\tau^2}G(\omega)\,d\omega. \tag{3.115}$$

- the integral function, determined by the energy spectrum of transmission factor $K_{sp}(\tau)$:

$$G(\omega)=\frac{2}{\pi}\int_0^\infty R(\tau)\cos\omega\tau\,d\tau; \qquad (3.116)$$

$$R(\tau)=\langle[K_{\text{вр}}(t)-K_{\text{вр}_0}][K_{\text{вр}}(t+\tau)-K_{\text{вр}_0}]\rangle=\sigma^2\rho(\tau); \qquad (3.117)$$

$$\sigma^2=\langle[K_{\text{вр}}(t)-K_{\text{вр}_0}]^2\rangle; \qquad (3.118)$$

$$K_{\text{ф}}=\langle K_{\text{ф}}(t)\rangle=K_{\text{вр}_0}K. \qquad (3.119)$$

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From the examination of expressions (3.107), (3.108), (3.113) and (3.114) it follows that under specific conditions <z(t)> and $\sigma_z^2(t)$ can with an increase in argument t unlimitedly grow/rise, i.e., the system of the 1st order with the random transmission factor can prove to be unstable. As it will be shown below, this occurs as a result of the assumption about the fact that $K_{\text{вр}}(t)$ - normal random function.

The analysis of expressions (3.107) and (3.113) shows that the mathematical expectation of the measuring errors of signal $x_{\text{д}}(t)$ has finite quantity, if

$$\frac{1}{2}\frac{K_{\text{ф}}}{K^2_{\text{вр}0}}\tau a(\tau)<1 \text{ при } 0<\tau<\infty. \qquad (3.120)$$

Key: (1). with.

On the other hand, from (3.108) and (3.114) it follows that the variance of error is final, if

$$\frac{K_0}{K^2_{sp0}}\left[\frac{\tau^2}{\tau+\nu}a(\tau)+\frac{\nu^2}{\tau+\nu}a(\nu)-\frac{1}{2}\cdot\frac{(\tau-\nu)^2}{\tau+\nu}a(\tau-\nu)\right]<1, \tag{3.121}$$

where τ and ν vary from 0 to ∞.

It is not difficult to note that condition (3.121) is more general/more common/more total, since inequality (3.120) ensues/escapes/flows out from (3.121) if arguments τ or ν are equal to zero. Inequality (3.121) is made most rigid in the case when $\tau=\nu$ accepts the form

$$\frac{K_0}{K^2_{sp0}}\tau a(\tau)<1. \tag{3.122}$$

Since a(τ) final function for any final ones τ, satisfaction of stability condition should be checked when $\tau\rightarrow\infty$. From (3.115) it follows that

$$\lim_{\tau\rightarrow\infty}\tau a(\tau)=\pi G(0),$$

Then expression (3.122) can be recorded as

$$\pi\frac{K_0}{K^2_{sp0}}G(0)<1. \tag{3.123}$$

Inequality (3.123) is the stability condition of the servo system of the 1st order with the randomly changing transmission factor.

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The reason for the instability of such linear parametric systems lies in the fact that for the systems with the transmission factor, distributed according to assumption according to the normal law with average/mean value $K_{вр0}$ and by dispersion σ^2, there are always random time intervals when the amplification of system is negative, which indicates the appearance of positive feedback.

In actuality in such systems as complex range-only radar, radio direction finders, transmission factor cannot be negative. Therefore the approximation of the law of the distribution of coefficient $K_{вр}(t)$ with the the large σ^2 by normal law is inadmissible.

§3.5. Errors of integrated system with the random transmission factor, which has the uniform spectrum in the limits of its passband.

In the majority the integrated systems of automatic radio equipment are very narrow-band; therefore it is expedient to consider the case when transmission factor $K_{вр}(t)$ has the continuous uniform spectrum in the limits of the passband of system, i.e., spectrum $K_{вр}(t)$ is approximated by the white noise:

$$G(\omega) = N_0,$$

where N_0 - intensity of white noise.

In this case function $a(\tau)$, determined by equation (3.115), takes the form

$$a(\tau)=\frac{N_0}{|\tau|}. \quad (3.124)$$

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Taking into account (3.124), the expression of the mathematical expectation of weight function $g_3(t, \tau)$ and average/mean value from the square of weight function of system respectively can be recorded as

$$\langle g_3(t, t-\tau)\rangle=\exp\{-(K_0-\delta_0)\tau\}, \quad (3.125)$$

$$\langle g_3(t, t-\tau) g_3(t, t-\nu)\rangle=\exp\{-K_0(\tau+\nu)+ +\delta_0(2\tau+2\nu-|\tau-\nu|)\}, \quad (3.126)$$

where

$$\delta_0=\frac{\pi}{2}\cdot\frac{K_0^2}{K_{\text{вр}0}^2}N_0 \quad (3.127)$$

- parameter, proportional to the intensity of the fluctuations of transmission factor.

In accordance with (3.106) and (3.125) the average/mean value of the measuring errors of range is equal to:

$$\langle z(t)\rangle = x_0 e^{-(K_0-\delta_0)t} + \frac{v}{K_0-\delta_0}\left[1 - e^{-(K_0-\delta_0)t}\right]. \quad (3.128)$$

In the steady-state mode/conditions

$$\langle z_y\rangle = \frac{v}{K_0-\delta_0}.$$

It is easy to see that in the case of system with the constant parameters, i.e., when $K_{вр}(t)=K_{вр0}$, we will obtain known expression $\frac{v}{K_0}$.

On the basis (3.128) with respect to the average/mean value of the measuring errors of input signal $x_{д}(t)$ by parametric system it is possible to say that the fluctuations of the transmission factor of system cause an increase in the average/mean value of the errors of system in comparison with the errors of system with the constant parameters.

Thus, this parametric system, which has the average/mean value of transmission factor K_0, in terms of the average/mean value of errors is equivalent to the servo system with the constant parameters, which has the transmission factor, equal to value $(K_0-\delta_0)$.

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The variance of error of the measurement of signal $x_{\text{д}}(t)$ taking into account adopted designations (3.127) for different components is defined as:

$$\sigma_{11}^2(t) = x_0^2 < g_3^2(t,0) > - x_0^2 [< g_3(t,0) >]^2 =$$
$$= x_0^2 [e^{-2(K_0 - 2\delta_0)t} - e^{-2(K_0-\delta_0)t}],$$

$$\sigma_{12}^2(t) = v^2 \int_0^t \int_0^t \exp\{-K_0(\tau+\nu) + \delta_0(2\tau + 2\nu -$$
$$- |\tau - \nu|)\} d\nu d\tau - v^2 \left[\int_0^t \exp\{-(K_0 - \delta_0)\tau\} d\tau\right]^2 =$$
$$= \frac{v^2\delta_0}{(K_0-\delta_0)^2(K_0-2\delta_0)} - \frac{4v^2\delta_0}{(K_0-\delta_0)^2(K_0-3\delta_0)} \times$$
$$\times \exp\{-(K_0-\delta_0)t\} + \frac{v^2}{(K_0-3\delta_0)(K_0-2\delta_0)} \times$$
$$\times \exp\{-2(K_0-2\delta_0)t\} - \frac{v^2}{(K_0-\delta_0)^2} \exp\{-2(K_0-\delta_0)t\}.$$

In the steady-state mode/conditions

$$\sigma_{12y}^2 = \frac{\delta_0}{K_0 - 2\delta_0} \cdot \frac{v^2}{(K_0-\delta_0)^2}. \qquad (3.129)$$

$$\sigma_{13}^2(t) = R_3(0) - 2(K_0-\delta_0)\int_0^t e^{-(K_0-\delta_0)\tau} R_3(\tau) d\tau +$$
$$+ (K_0-\delta_0)(K_0-3\delta_0)\int_0^t\int_0^t \exp\{-K_0(\tau+\nu) +$$
$$+ \delta_0(2\tau + 2\nu - |\tau-\nu|)\} R_3(\tau-\nu) d\tau d\nu; \qquad (3.130)$$

$$\sigma_2^2(t) = K^2 \int_0^t\int_0^t \exp\{-K_0(\tau+\nu) +$$
$$+ \delta_0(2\tau+2\nu - |\tau-\nu|)\} R_f(\tau-\nu) d\tau d\nu;$$

$$\sigma_3^2(t) = \int_0^t\int_0^t \exp\{-K_0(\tau+\nu) +$$
$$+ \delta_0(2\tau+2\nu-|\tau-\nu|)\} R_\varphi(\tau-\nu) d\tau d\nu.$$

As an example let us consider the case when $\varphi_{\pi}(t)$ - normal stationary process with the correlation function of the form

$$R_{\varphi}(\tau)=\sigma_{w}^{2}\exp(-\alpha_{w}|\tau|).$$

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In this case the variance of error of the measurement of signal of those caused by interferences along the channel of self-contained meter, is written/recorded as

$$\sigma_{3}^{2}(t)=\frac{\sigma_{w}^{2}}{(K_{o}-3\delta_{o}-\alpha_{w})(K_{o}-\delta_{o}+\alpha_{w})}-$$
$$-\frac{2\sigma_{w}^{2}}{(K_{o}-3\delta_{o}-\alpha_{w})(K_{o}-\delta_{o}+\alpha_{w})}\exp\{-(K_{o}-\delta_{o}+\alpha_{w})t\}+$$
$$+\frac{\sigma_{w}^{2}}{(K_{o}-3\delta_{o}-\alpha_{w})(K_{o}-\delta_{o}+\alpha_{w})}\times$$
$$\times\exp\{-2(K_{o}-2\delta_{o})t\}-\frac{\sigma_{w}^{2}}{2(K_{o}-3\delta_{o}+\alpha_{w})(K_{o}-2\delta_{o})}+$$
$$+\frac{\sigma_{w}^{2}}{2(K_{o}-3\delta_{o}-\alpha_{w})(K_{o}-2\delta_{o})}\exp\{-2(K_{o}-2\delta_{o})t\}+$$
$$+\frac{\sigma_{w}^{2}}{2(K_{o}-\delta_{o}+\alpha_{w})(K_{o}-2\delta_{o})}-$$
$$-\frac{\sigma_{w}^{2}}{2(K_{o}-3\delta_{o}+\alpha_{w})(K_{o}-2\delta_{o})}\exp\{-2(K_{o}-2\delta_{o})t\}.$$

In the steady-state mode/conditions:

$$\sigma_{3y}^{2}=\frac{\sigma_{w}^{2}}{(K_{o}-2\delta_{o})(K_{o}-\delta_{o}+\alpha_{w})}. \qquad (3.131)$$

From (3.131) it is evident that the variance of error, caused by errors in the channel of self-contained meter, parametric systems are more than matching system analogous to dispersion with the constant parameters. Dispersion σ_{3}^{2} falls from an increase in the operating speed of system, i.e., with increase of K_{o}.

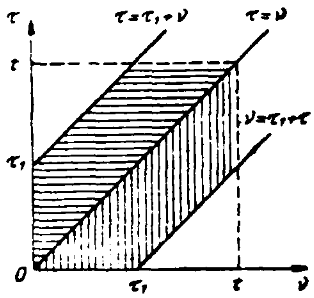

Fig. 3.8.

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During the computation of the variance of error of the measurement of signal $x_{\text{п}}(t)$, of those caused by interferences on the radio channel, the correlation function of additive noise $f_p(t)$ is represented in the form [see application/appendix, formula (P.25)].

$$R_f(\tau)=\begin{cases} \sigma_f^2\left(1-\frac{|\tau|}{\tau_1}\right) & \text{(1) при } |\tau|\leq\tau_1, \\ 0 & \text{(1) при } |\tau|>\tau_1. \end{cases}$$

Key: (1). with.

where σ_f^2 - dispersion of interferences at the output of discriminator; τ_1 - duration of disturbing pulses at the output of discriminator.

Then we have;

$$\sigma_2^2(t) = K^2 \int_0^t \int_0^t \exp\{-K_0(\tau + \nu) + \delta_0(2\tau + 2\nu - |\tau - \nu|)\} \sigma_f^2 \left(1 - \frac{|\tau - \nu|}{\tau_1}\right) d\tau\, d\nu \ \text{при}^{(1)}\ |\tau - \nu| < \tau_1.$$

Key: (1). with.

Range of integration during determination $\sigma_2^2(t)$ is represented in Fig. 3.8 (shaded part). In accordance with this the integral is equal to:

$$\sigma_2^2(t) = K^2 \sigma_f^2 \int_0^t d\nu \int_0^\nu \exp\{-K_0(\tau + \nu) + \delta_0(3\tau + \nu)\} \left(1 - \frac{\nu - \tau}{\tau_1}\right) d\tau - K^2 \sigma_f^2 \int_{\tau_1}^t d\nu \int_0^{\nu - \tau_1} \exp\{-K_0(\tau + \nu) + \delta_0(3\tau + \nu)\} \left(1 - \frac{\nu - \tau}{\tau_1}\right) d\tau + K^2 \sigma_f^2 \int_0^t d\nu \int_\nu^t \exp\{-K_0(\tau + \nu) + \delta_0(\tau + 3\nu)\} \left(1 - \frac{\tau - \nu}{\tau_1}\right) d\tau - K^2 \sigma_f^2 \int_0^{t - \tau_1} d\nu \int_{\tau_1 + \nu}^t \exp\{-K_0(\tau + \nu) + \delta_0(\tau + 3\nu)\} \left(1 - \frac{\tau - \nu}{\tau_1}\right) d\tau.$$

After fulfilling the appropriate transformations, we will obtain

$$\sigma_{2y}^2 = K^2 \sigma_f^2 \frac{\tau_1(K_0 - \delta_0) - 1 + e^{-(K_0 - \delta_0)\tau_1}}{\tau_1 (K_0 - \delta_0)^2 (K_0 - 2\delta_0)}.$$

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Since usually is satisfied the condition $\tau_1(K_e-\delta_e)<<1$,

$$\sigma_{2y}^2=\frac{\sigma_f^2}{K_{sp0}^2}\cdot\frac{\tau_1}{2(K_e-2\delta_e)}K_0^2. \qquad (3.132)$$

Variance of error σ_{2y}^2 grows with an increase in the average transmission factor of system K_e and intensity of fluctuations δ_e.

In accordance with the expressions for signal and dispersion of interferences at the output of temporary/time discriminator (P.19) and (P.20) when $\tau_{и}=\tau_{сел}$ (see the appendix) the variance of error of the measurement of signal $x_д(t)$, of those caused by interferences on the radio channel, in the trimmed/steady-state mode/conditions it is equal to:

$$\sigma_{2кy}^2=\frac{c_1 k_{1п}^2 \tau_и T K_0^2}{8\Delta f_{пр}(K_e-2\delta_e)}\cdot\frac{\sqrt{2}+2<q_0^2(t)>}{[<q_0^2>]^2}-$$

- for the case of square-law detection;

$$\sigma_{2лy}^2=\frac{c_1 k_{1п}^2 \tau_и T K_0^2}{8\Delta f_{пр}(K_e-2\delta_e)}\times$$

$$\times\frac{\sqrt{2}+1.3<q_0^2(t)>-0.5<q_0^4(t)>}{[<q_0^2(t)>]^2}-$$

- for the case of linear detection with $<g_e(t)><1$.

In accordance with (3.130) the expression for the dispersion, caused by the component $\beta(t)$, in the trimmed/steady-state mode/conditions is equal to:

$$\sigma_{13y}^{2}=\sigma_{\beta}^{2}\frac{K_{0}\alpha_{\lambda}-2\delta_{0}\alpha_{\lambda}-K_{0}\delta_{0}+\delta_{0}^{2}}{(K_{0}-2\delta_{0})\,(K_{0}-\delta_{0}+\alpha_{\lambda})}. \qquad (3.133)$$

For an example it was here assumed that

$$R_{\beta}(\tau)=\sigma_{\beta}^{2}\,e^{-\alpha_{\lambda}|\tau|}.$$

From expression (3.133) follows that the greater the operating speed of system, i.e., the more K_0, the less the variance of error of the measurement of useful signal.

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The general/common/total expression for the variance of error of the measurement of signal $x_{\text{п}}(t)$ in the trimmed/steady-state mode/conditions with the approximation of the fluctuations of transmission factor $K_{\text{вр}}(t)$ by white noise it is possible to record in the form:

$$\begin{aligned}\sigma_{\varepsilon y}^{2}&=\sigma_{12y}^{2}+\sigma_{13y}^{2}+\sigma_{2y}^{2}+\sigma_{3y}^{2}=\\&=\frac{\delta_{0}\sigma^{2}}{(K_{0}-2\delta_{0})\,(K_{0}-\delta_{0})^{2}}+\\&+\frac{\sigma_{\beta}^{2}\,(K_{0}-2\delta_{0})\,\alpha_{\lambda}-\delta_{0}\,(K_{0}-\delta_{0})}{(K_{0}-2\delta_{0})\,(K_{0}-\delta_{0}+\alpha_{\lambda})}+\\&+\frac{\sigma_{f}^{2}\,\tau_{1}\,K_{0}^{2}}{2K_{\text{вр}0}^{2}\,(K_{0}-2\delta_{0})}+\frac{\sigma_{u}^{2}}{(K_{0}-2\delta_{0})\,(K_{0}-\delta_{0}+\alpha_{u})}.\end{aligned} \qquad (3.134)$$

Taking into account (3.128) and (3.134) the expression for the average/mean value from the square of the measuring error of useful signal $x_3(t)$ in the trimmed/steady-state mode/conditions can be recorded as

$$<z_y^2>=[<z_y>]^2+\sigma_{zy}^2. \qquad (3.135)$$

where $<z_y>$ it is determined in accordance with (3.128).

After setting in (3.135) the parameter δ_0, which is determining the intensity of the fluctuations of transmission factor, equal to zero, we will obtain expression known earlier for the errors of integrated system without taking into account the fluctuations of the parameters of system.

With the fluctuating transmission factor $K(t)$ the problem of guaranteeing the minimum of the average/mean value from the square of the measuring errors of signal $x_3(t)$ is reduced to the determination of the optimum average transmission factor of system K_0. The optimum value K_0 is found from the condition of equality to zero partial derivative

$$\frac{\partial}{\partial K_0}<[z(t)]^2>=0.$$

On the basis (3.123) it is possible to obtain the stability

condition of parametric system for the case when the fluctuations of transmission factor are approximated by the white noise:

$$2\delta_0 < K_0. \qquad (3.136)$$

Let us note that the obtained stability condition (3.136) directly escape/ensues from the examination of expression for the variance of error of the measurement of signal $x_{\text{п}}(t)$.

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S3.6. Errors of integrated system with the random transmission factor, which has the nonuniform spectrum in the limits of its passband.

When the fluctuation of the transmission factor of system cannot be approximated by white noise, formulas for the statistical characteristics of the errors in their majority through the elementary functions cannot be expressed. Thus, for instance, let the fluctuations of the transmission factor of temporary/time discriminator $\dot{K}_{\text{вр}}(t)$ have the spectral density of the form very frequently encountered in practice:

$$G(\omega) = \frac{4\alpha\sigma^2}{(\alpha^2 + \omega^2)}. \qquad (3.137)$$

Then function a(τ), determined by dependence (3.115), is equal

to

$$a(\tau)=\frac{2\sigma^2}{\alpha\tau^2}\left[|\tau|-\frac{1}{\alpha}(1-e^{-\alpha|\tau|})\right] \qquad (3.138)$$

Taking into account (3.138) the expressions for the average/mean value of weight function $g_3(t, \tau)$ and the average/mean value from the square of this weight function respectively *can be recorded:*

$$<g_3(t, t-\tau)>=e^{-\eta d^2}\exp\{-K_0(1-\eta d)\tau+\eta d^2 e^{-\alpha\tau}\},$$
$$(3.139)$$
$$<g_3(t, t-\tau)g_3(t, t-\nu)>=\exp\{-K_0(\tau+\nu)+2\eta d^2\alpha\tau+$$
$$+2\eta d^2\alpha\nu+2\eta d^2 e^{-\alpha\nu}-\eta d^2\alpha|\tau-\nu|-$$
$$-\eta d^2 e^{-\alpha|\tau-\nu|}-3\eta d^2\}, \qquad (3.140)$$

where $\eta=\frac{\sigma^2}{K^2_{\text{вр}0}}$ - ratio of the dispersion of function $K_{\text{вр}}(t)$ to the square of its average/mean value: $d=\frac{K_0}{\alpha}$ - the ratio of the average transmission factor of system to the attenuation factor of fluctuation $K_{\text{вр}}(t)$.

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In accordance with (3.139) the average/mean value of the measuring errors of range is equal

$$<z(t)>=x_0 e^{-\eta d^2}\exp\{-K_0(1-\eta d)t+$$
$$+\eta d^2 e^{-\alpha t}\}+v\int_0^t e^{-\eta d^2}\exp\{-K_0(1-\eta d)\tau+$$
$$+\eta d^2 e^{-\alpha\tau}\}d\tau.$$

In the trimmed/steady-state mode/conditions the average/mean

value of the errors is defined as

$$\langle z_y \rangle = \frac{v}{K_0} \cdot \frac{e^{-\eta d}}{1-\eta d} \Phi(d - d^2\eta_1, d - \eta_1 d^2 + 1, \eta_1 d^2).$$

where

$$\Phi(x, y, z) = 1 + \frac{x}{y} \cdot \frac{z}{1!} + \frac{x(x+1)}{y(y+1)} \cdot \frac{z^2}{2!} + \dots$$

- degenerate hypergeometric function.

For the majority of the practical cases it suffices to consider the first two or three members of series/row; therefore need in the use of the tables of the degenerate hypergeometric functions is eliminated.

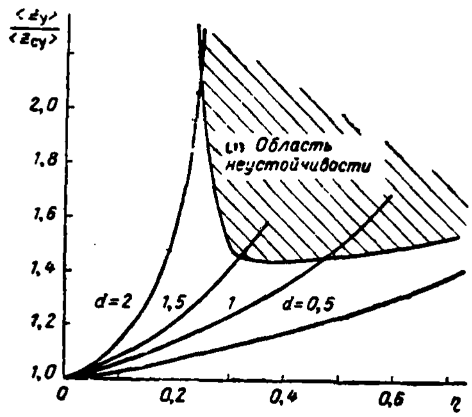

Fig. 3.9.

Key: (1). Unstable region.

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Fig. 3.9 presents the dependences of the ratio of conservative value of the mathematical expectation of the measuring errors of signal $x_{\text{н}}(t)$ to conservative value of the mathematical expectation of matching system with constant parameters $\langle z_{cy} \rangle = \frac{v}{K_0}$ from η at different values of d. Curves show that $\langle z_y \rangle$ is always more than $\langle z_{cy} \rangle$. Value $\langle z_y \rangle$ grows with increase η. i.e. with an increase in the dispersion of fluctuations of function $K_{\text{пп}}(t)$. With the decrease of the attenuation factor of fluctuations α value $\langle z_y \rangle$ also grows.

The variance of error of the measurement of signal $x_a(t)$ taking into account (3.138) for different components is defined as

$$\sigma_{11}^2(t)=x_0^2\exp\{-2K_0(1-2\eta d)t+4\eta d^2 e^{-\alpha t}-$$
$$-4\eta d^2\}-x_0^2\exp\{-2K_0(1-\eta d)t+2\eta d^2 e^{-\alpha t}-2\eta d^2\},\qquad(3.141)$$

$$\sigma_{12}^2(t)=v^2\int_0^t\int_0^t\exp\{-K_0(\tau+\nu)+2\eta d^2\alpha\tau+$$
$$+2\eta d^2 e^{-\alpha\tau}+2\eta d^2\alpha\nu+2\eta d^2 e^{-\alpha\nu}-\eta d^2\alpha|\tau-\nu|-$$
$$-\eta d^2 e^{-\alpha|\tau-\nu|}-3\eta d^2\}\,d\tau\,d\nu-v^2\left[e^{-\eta d^2}\int_0^t\exp\{-K_0\times\right.$$
$$\left.\times(1-\eta d)\tau+\eta d^2 e^{-\alpha\tau}\}\,d\tau\right]^2,\qquad(3.142)$$

$$\sigma_2^2(t)=\int_0^t\int_0^t\sigma_f^2K^2\exp\{-K_0(\tau+\nu)+2\eta d^2\alpha\tau+2\eta d^2 e^{-\alpha\tau}+$$
$$+2\eta d^2\alpha\nu+2\eta d^2 e^{-\alpha\nu}-\eta d^2\alpha|\tau-\nu|-\eta d^2 e^{-\alpha|\tau-\nu|}-$$
$$-3\eta d^2\}\left(1-\frac{|\tau-\nu|}{\tau_1}\right)d\tau\,d\nu;\qquad(3.143)$$

$$\sigma_3^2(t)=\sigma_w^2\int_0^t\int_0^t\exp\{-K_0(\tau+\nu)+2\eta d^2\alpha\tau+2\eta d^2 e^{-\alpha\tau}+$$
$$+2\eta d^2\alpha\nu+2\eta d^2 e^{-\alpha\nu}-\eta d^2\alpha|\tau-\nu|-\eta d^2 e^{-\alpha|\tau-\nu|}-$$
$$-3\eta d^2\}\exp\{-\alpha_w|\tau-\nu|\}\,d\tau\,d\nu.\qquad(3.144)$$

Integrals in expressions (3.141)-(3.144) through the elementary functions are not expressed; therefore during the computation of the statistical characteristics of errors one should resort to the method of numerical integration.

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Some results of the calculations of average/mean value and variance of error of measurement of the signal $x_{\text{д}}(t)$, obtained with the use of computer, are represented in Fig. 3.10-3.14. The examination of curves in the given figures shows that with an increase in the intensity of the fluctuations of the transmission factor η of the measuring error of signal $x_{\text{д}}(t)$ they grow; moreover it is slower than the fluctuation, the greater the error. On the more high speed system of the fluctuation of transmission factor is had an more essential effect. Consequently, the integrated systems of automation the fluctuations of the parameters are manifested to the considerably smaller degree, than simple systems.

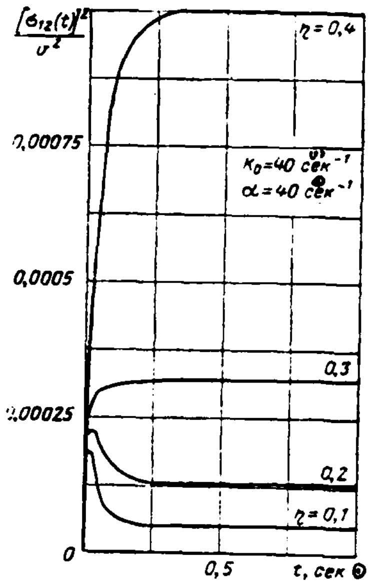

Fig. 3.10.

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From the examination of curves in Fig. 3.10 it is evident that with an increase in the average transmission factor of system K_0 the variance of error of the measurement of signal $x_{\text{п}}(t)$, of those caused by interferences on the radio channel, increases.

On the basis (3.123) it is possible to obtain the following expression for the stability condition of the parametric system of the 1st order in the case when the fluctuations of transmission factor $K_r(t)$ have a spectral density of type (3.137):

$$\eta d < 0,5.$$

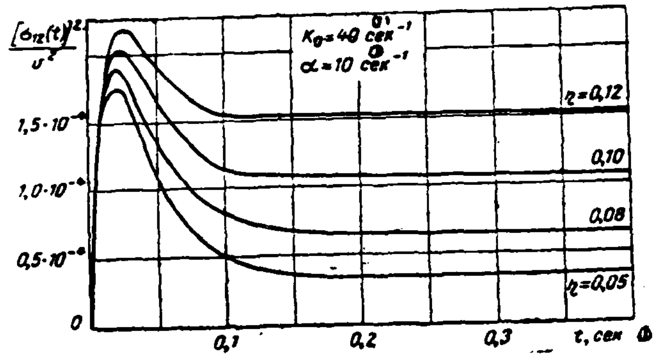

Fig. 3.11.

Key: (1). s.

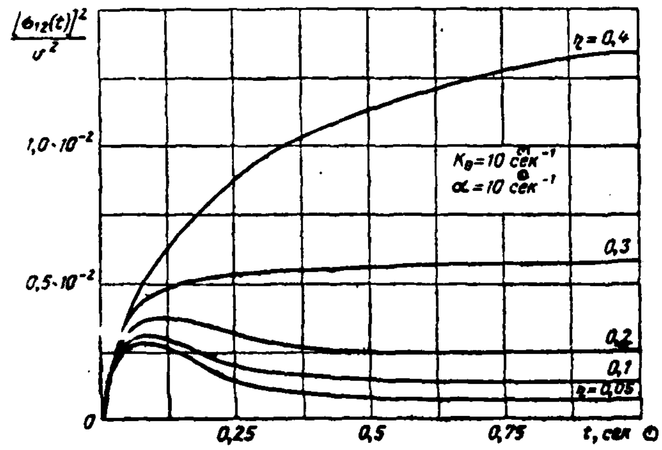

Fig. 3.12.

Key: (1). s.

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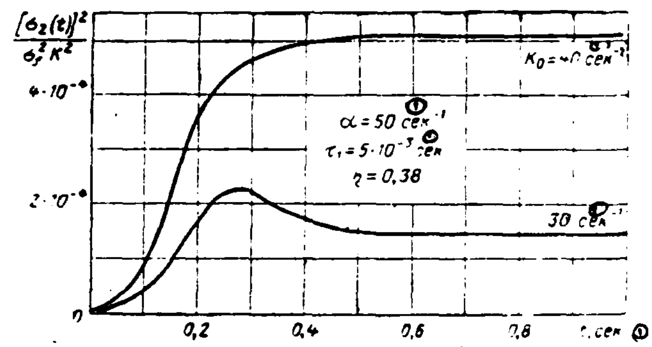

Fig. 3.13.

Key: (1). s.

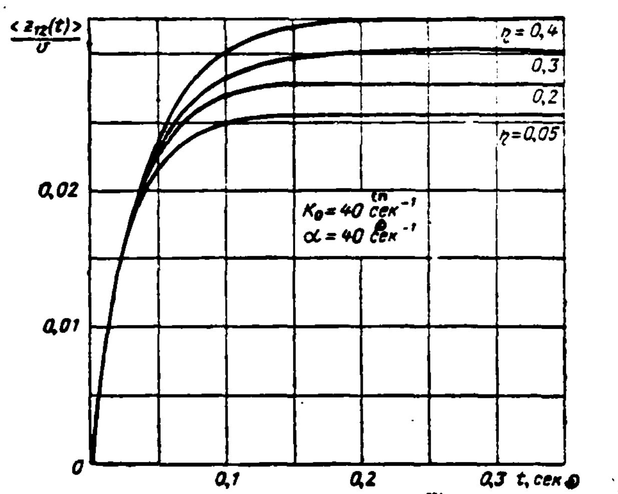

Fig. 3.14.

Key: (1). s.

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Thus, the fluctuations of the transmission factor of the integrated systems of automatic radio equipment cause a decrease in the operating speed of systems and an increase in the fluctuation errors. As the average/mean, so fluctuation errors of parametric systems are the greater, the higher the intensity of the fluctuations of transmission factor $K_t(t)$, the slower in the time it it is changed. The fluctuations of the parameters produce considerably smaller errors in the integrated systems of automatic radio equipment in comparison with simple systems, as a result of the lower speed along the radio engineering channel in integrated systems.

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Chapter 4.

INTEGRATED AUTOMATIC GONIOMETRIC SYSTEMS.

Goniometric devices/equipment are used in different radio engineering stations. In this chapter are examined the methods of the aggregation of the radio engineering servo goniometers with independent meters of angular oscillations of the flight vehicles, on which is installed goniometrical devices/equipment. These methods are examined mainly in connection with aircraft target-tracking radar and to the radio engineering caps of homing missiles.

The analysis of integrated goniometric systems has the purpose of systematizing available material, to consider some special features/peculiarities of the construction of such systems, and to also illustrate the basic condition/positions, examined in the previous chapters.

The aggregation of the radio engineering servo goniometers can be realized not only for the purpose of the decrease of effect on their work of angular oscillations of flight vehicle. To be measured

and to be introduced into the servo system can also the signals, caused by the displacement/movement of flight vehicle in the space. Here these questions conscious are omitted. They are examined in Chapter 6 in connection with the goniometrical channel of the azimuth and range finding system of short-range navigation.

Is analyzed below the mode/conditions of target tracking. However, all diagrams in question can be used for purposes of the stabilization of antenna systems in the search mode for target.

S4.1. Input effects and the special features/peculiarities of the goniometrical channels of homing systems.

The data carrier about angular target position is the signal going from it. Information is included in the amplitude, the phase or the frequency of received signal.

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The direction-finding device/equipment, which forms part of goniometer, extracts this information from the radio signal and converts it into the electrical signal, proportional to the angle between the direction to the fir tree and the equisignal direction. During the study of the dynamics of goniometrical channels it is

possible to be distracted from the method of direction-finding and as the input effects to examine angles themselves.

Input effects are caused by the relative motion of target and flight vehicle[1], and also by the angular motion of foundation, on which is established/installed the antenna.

FOOTNOTE [1]. Subsequently, where this is specially not stipulated, we will call, for brevity, the flight vehicle a rocket. ENDFOOTNOTE.

The relationships/ratios between the angles, characteristic for the work of the goniometrical channel of homing system in one of the control planes, are illustrated by Fig. 4.1, where target positions Ts and equisignal direction (RSN) in fixed coordinate system xoy are characterized by angles ε and ε_{K} respectively, and the position of axis of rocket ox_1 - by angle ϑ; $w_{\text{пр}}$ - relative velocity of rocket and target.

The problem of goniometrical channel is of the tracking the equisignal direction of direction to the target with the permissible error θ. be reached this can by different methods.

Equisignal direction can be rigidly connected with the axis of rocket, i.e., antenna can be fixed relative to missile body. Then $\gamma=\theta$

and the input value of goniometer is the angle

$$\theta = \varepsilon - \vartheta. \qquad (4.1)$$

The tracking of direction to the target with the fixed antenna is realized due to the rotation of the axis of rocket itself or aircraft.

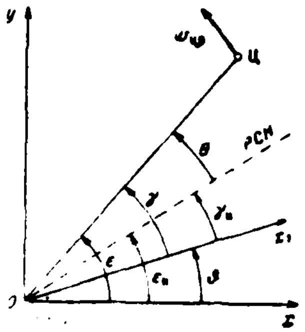

Fig. 4.1.

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In the goniometrical devices/equipment of another type equisignal direction is tracked with the help of the separate servo system and as the input effect must be examined the angle

$$\gamma = \varepsilon - \vartheta, \tag{4.2}$$

and θ is following error.

The output signals of goniometer depending on the method of induction utilized in the system are the measured values of angle γ, angle ε or $\dot{\varepsilon}$. The signals, utilized for the rocket control or for the construction of lead angle in RLS of aircraft, in contrast to the output signals of coordinator, we will subsequently call controlling. Regardless of the fact what signal is measured, goniometer must reliably track a target, i.e., track changes in both angle ϑ and

angle ε.

The fact indicated forces during the error analysis of the goniometrical channels of the homing systems, which use as control signals ε or $\dot{\varepsilon}$, to be interested not only measuring errors, but also in tracking errors θ.

Rate of change in the angle ϑ, as a rule, exceeds rate of change in angle ε. Even for the aerial target, especially during the large removal/distance of rocket from the target, the instantaneous values of derived input values $\dot{\varepsilon}(t)$ and $\dot{\vartheta}(t)$ can differ several times. But if we take the target fixed or relative to low-mobility, then it will seem that angular velocities $\dot{\varepsilon}(t)$ and $\dot{\vartheta}(t)$ can differ into ten and even into hundreds of times.

An increase in the dynamic accuracy of servo system requires the expansion of its passband, and this, as already mentioned, it contradicts the requirement of an increase in the freedom from interference. For the goniometrical channels the expansion of the passband of servo system of more than the specific value leads not to an increase, but to the decrease of the accuracy of the measurement of angular coordinates, since even in the absence of outside interferences with the fluctuation of the signal echo from the target on the phase or amplitude is produced further random error. This

error proves to be the greater, the wider the passband of servo system.

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The misalignments of rockets (change in the angle ϑ) during echo signal fading can lead to the loss of target tracking on the angles, since the servo system of goniometer proves to be extended or has too low a transmission factor.

The requirement of the maximum decoupling of coordinator from the angular oscillations of rocket in the control systems, which use as the control signals, proportional to the angular velocity of line rocket - target, is dictated also by the dynamics of the duct/contour of guidance.

If angle ϑ is mastered by system with the errors, then can arise undesirable phenomena connected with the possibility of the loss of stability by the control system.

Usually the control signal which subsequently acts on the autopilot of rocket, is the angular velocity $\dot{\varepsilon}$ of line of fire (see [5]) or angle itself ε.

The very simplified functional diagram of control system by flight vehicle is depicted in Fig. 4.2. The designation/purpose of the system of control (with the direct guidance method) consists of the continuous coincidence of the axis of rocket with the direction to the target.

Radio engineering coordinator tracks angle γ. For obtaining control signal to angle $\gamma_{н}$ with the direct guidance method is supplemented the angle ϑ (for example, from the gyroscope) and signal $\varepsilon=\gamma_{н}+\vartheta$ is supplied to the system of rocket control, producing the rotation of its axis to angle $\gamma_{н}$. The latter varies until axis of rocket is directed to the target, i.e., it will be achieved/reached equality $\varepsilon=\vartheta$.

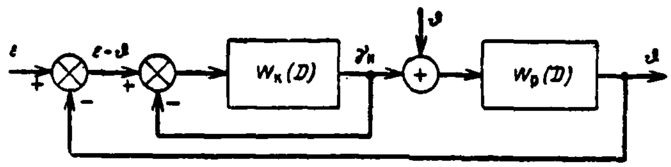

Fig. 4.2.

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If the transfer function of radio coordinator is designated through $\Phi_к(D)$, and rocket $W_p(D)$, for the output angle of homing system it is possible to record

$$\vartheta = (\varepsilon - \vartheta) W_p(D)\, \Phi_к(D) + \vartheta\, W_p(D),$$

or

$$\vartheta = \frac{\Phi_к(D)\, W_p(D)\, \varepsilon(t)}{1 + W_p(D)\, [\Phi_к(D) - 1]}. \qquad (4.3)$$

From the obtained expression it is evident that the system contains both the positive and negative feedback on ϑ.

The presence of positive connection/communication in the control loop places difficult to achieve conditions on the identification of the parameters of the servo system of coordinator.

As it follows from expression (4.3), satisfaction of stability

condition is difficult. Radio engineering coordinator works on the fluctuating signal and with the amplitude methods of direction finding is system with the random amplification factor. Therefore during any selection of the average/mean value of the factor of amplification of signal fading they will convert system into some the moment of time into the unstable. One ought not, of course, to forget that besides technical difficulties, the expansion of the passband of coordinator contradicts the requirement of an increase in freedom from interference. The facts indicated caused wide application in the homing systems, which use as a control signal, proportional to the velocity of the motion of line rocket - target, gyroscopic coordinators. In the interceptor radar is found use of a diagram of other types [14].

Tendency to throttle/taper the band of goniometrical channel without the loss of dynamic accuracy was necessary the development of the methods of stabilization of angular antenna positions relative to the fixed coordinate system.

By the stabilization of angular antenna position it succeeds, although partially, to reduce the effect of the component of the input effect, caused by the oscillations of the angular position of rocket or aircraft.

To solve the problem indicated is possible, using a separate meter of the angular displacements of rocket (aircraft), and the following system to construct as the integrated measuring system of angular coordinates.

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It is convenient to distinguish in this case two ducts/contours (two systems): the duct/contour of the stabilization (actually the system of the compensation for the angular oscillations of the rockets ϑ which previously were designated through x_a) and the tracking circuit (system of the final adjustment of input effect γ).

In connection with goniometers with equisignal direction fixed relative to the axis of rocket it cannot be spoken about the decoupling of motions of RSN and axis of rocket. However, the essence of matter from this does not vary. The measurement of the angular oscillations of rocket by the separate meter (ISD) and introduction to control loop of the rocket of the signals, proportional to angle ϑ or $\dot{\vartheta}$, is accomplished with the same target, as in the servo goniometers.

There are many different methods of the solution of the problem indicated. But independent of the utilized method of the construction of the integrated measuring system of angular coordinates, it must provide on one hand, the high degree of the decoupling of the motion

of antenna and axis of rocket (in the mobile coordinator), and on the other hand, the high accuracy of the tracking of direction to the target.

Together with the stabilization systems of antennas are applied and the so-called stabilization systems of data. In the systems of this type the motions of antenna are not untied from the motions of the axis of rocket, but before the input of the signals, obtained with the help of the automatic direction finder into the autopilot, of them are subtracted the signals, proportional to the angular oscillations of the axis of rocket. The latter, naturally, are obtained with the help of the separate self-contained meter.

Although in the stabilization systems of data is not permitted the fundamental contradiction in the requirements of high dynamic accuracy and freedom from interference of servo system, the latter must be related to the class of complex meters and one example of its fulfillment will be examined below.

In the most general form the requirement for the integrated system is reduced to the measurement of one or the other angular coordinates of target with minimum accumulated error, caused by the angular displacements of target, by the angular oscillations of rocket, by the fluctuations of the signal, reflected from the target

during its reliable accompaniment. In a number of cases must also be considered natural or electronic jamming, which enter the input of system together with the useful signal.

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The selection of the method of stabilization of antenna system in the mode/conditions of accompaniment depends on the series/row of reasons. Decisive among them they are:

- sizes/dimensions of the antenna system:

- utilized law of the homing:

- required accuracy of target tracking.

Antenna system should be stabilized in two planes: in the vertical and the horizontal. If rocket is stabilized in the same planes, then the channels of stabilization prove to be untied. However, with the banks of rocket appears the connection/communication between the channels and the signals of the stabilization of antenna must by correspondingly be counted over before their input/introduction into the servo system.

As the self-contained sensors of the angular oscillations of rockets are used high-speed/high-velocity and displacement gyroscopes. The stabilization systems of antennas it is accepted to class according to the types of the utilized gyroscopes, the site of installation and the method of using their signals. A strict analysis of the passage of the signals through the coordinator requires the account of entire duct/contour of guidance of rocket (aircraft). However, taking into account that the passband of the radio engineering meter of angular coordinates is substantially wider than the band of the duct/contour of missile targeting, the effect of the latter on the equations for the signals of goniometer it is possible to disregard and to examine separately goniometer.

In conclusion let us again emphasize that angle is changed both due to the motion of target and due to forward motion of the object on which is established/installed goniometrical system.

Second component frequently can be measured and signal proportional to it is used for purposes of further increase in freedom from interference and accuracy of goniometrical system by its introduction to servo system in one of the methods examined above.

S4.2. Stabilization systems with the gyroscopic actuating elements.

It is most full the requirements, presented to the stabilization system of antenna, they are performed in gyroscopic type coordinators.

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An advantage of this type of coordinator is the capability technically simply and with sufficient accuracy to measure the angular velocity of line rocket - target.

As an example let us consider monogyroscope coordinator with the mechanical displacement of equisignal direction [5]. The functional diagram of device/equipment is given in Fig. 4.3. The free gyroscope, which has rotor 1, internal framework 2, external framework 3, is furnished with two correction engines 4 and 5. With the axis of gyroscope is connected the moving element of the antenna (reflector) A. Therefore of the fluctuation of the axis of rocket, characterizable by angle ϑ, do not have an effect on the position of the axis of antenna.

With the divergence of target from the equisignal direction (i.e. relative to axis $O_{H}x_{H}$) appear the signals at the outputs of two-channel direction-finding device/equipment 6. After the passage of the power amplifiers 7 and 8 they enter torque motors 4 and 5,

with the help of which are placed the moments/torques on the internal and external framework of gyroscope.

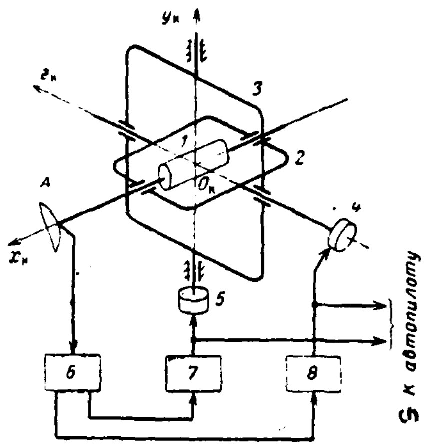

Fig. 4.3.

Key: (1). To the autopilot.

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The axis of gyrorotor will precess relative to axis $O_к y_к$ of the external framework of gyroscope under the action of the moment/torque, created by engine 4, and relative to axis $O_к z_к$ during the supplying of error voltage on engine 5. Gyro precession in both planes derives/concludes equisignal direction to the target and reduces the error signal to zero.

The analysis of the work of diagram is shown [5] that due to signal lag in the direction-finding device/equipment and presence of

axial moment inertia of gyroscope both channels of coordinator prove to be connected. However, during the proximate analysis or on the assumption that the disagreement/mismatch occurs only in one of the planes (for example, in plane $x_{\text{к}}O_{\text{к}}y_{\text{к}}$). it is possible to be bounded to the analysis of one channel.

Fig. 4.4 gives the block diagram of one channel of the coordinator. For the simplification the inertness of direction-finding device/equipment (in abbreviated form direction finder) and correction engine let us disregard/neglect, after assuming that $K_{\text{пу}}$ - transmission factor of direction-finding device/equipment; and amplifier and gyroscopic drive have transfer functions respectively:

$$\frac{K_{\text{у}}}{T_{\text{у}}D+1},\ \frac{K_{\text{гп}}}{D},$$

where $K_{\text{гп}}=J_{\text{дв}}/H$; $J_{\text{дв}}$ - moment of the inertia of rotor; H - the moment of momentum of gyroscope.

Similar type coordinators are included in the homing systems, which use as the control signal derivative of angle $\dot{\varepsilon}$. Control signals are removed/taken from the inputs of correction engines (signal $u_{\text{к}}$).

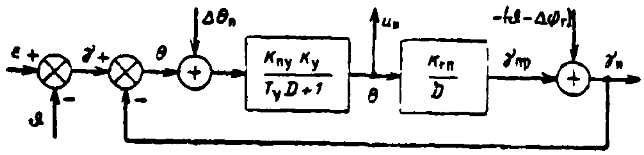

Fig. 4.4.

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The input effect of tracking system is angle $\gamma=\varepsilon-\vartheta$ between the axis of rocket and the direction to the target, which can be changed both as a result of the angular displacements of target and rocket relative to the fixed axis (angle ε), and as a result of the angular oscillations of rocket (angle ϑ). The output (measured) value of coordinator is angle $\gamma_н$ of the rotation of equisignal direction relative to the axis of rocket. The latter has two components: the precession angle of the framework of gyroscope ($\gamma_{пр}$), of that caused by the action of moment sensor, and the angle of rotation of the axis of gyroscope relative to the axis of rocket, caused by the fact that with a change in the angle ϑ the axis of gyroscope retains its direction in the space, i.e., is turned relative to the axis of rocket to the angle - ϑ. Since this rotation occurs with certain error (error of gyroscope $\Delta\psi_r$), from angle $\gamma_{пр}$ should be subtracted not angle ϑ, but angle $\vartheta-\Delta\psi_r$. Thus, the output (measured) value of

angle $\gamma_{\text{и}}$, which it masters coordinator, will be the angle

$$\gamma_{\text{и}} = \gamma_{\text{пр}} - (\vartheta - \Delta\psi_{\text{г}}).$$

Disagreement/mismatch in the system

$$\theta = \gamma - \gamma_{\text{и}}. \qquad (4.4)$$

All this is reflected on the structural diagram in Fig. 4.4, where is taken into consideration also the component of radio interferences, designated in the form of equivalent interference angle $\Delta\theta_{\text{п}}$.

The examination of figure shows that according to above classification accepted the diagram in question can be related to the class of integrated systems with the position correction.

Equation for the error of the tracking of input effect (θ) taking into account the action of radio interferences and fluctuations of the echo signal can be recorded and the form

$$\theta = \varepsilon - \vartheta - \gamma_{\text{и}} = \frac{1}{1 + W_{\text{к}}(D)}\varepsilon - \\ - \frac{W_{\text{к}}(D)}{1 + W_{\text{к}}(D)}\Delta\theta_{\text{п}} - \frac{1}{1 + W_{\text{к}}(D)}\Delta\psi_{\text{г}}, \qquad (4.5)$$

where

$$W_{\text{к}}(D) = \frac{K_{\text{ку}} K_{\text{у}} K_{\text{ги}}}{D(T_{\text{у}}D + 1)}$$

- transfer function of the extended system.

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After substituting in (4.5) the expression for $W_{\text{к}}(D)$, we will obtain

$$\theta=\frac{D(T_{y}D+1)}{N_{\text{к}}(D)}\varepsilon-\frac{K_{v}\Delta\theta_{n}}{N_{\text{к}}(D)}-\frac{T_{y}D+1}{N_{\text{к}}(D)}\Delta\psi_{\text{г}}, \qquad (4.6)$$

where $N_{\text{к}}(D)=T_{y}D^{2}+D+K_{v}$ – the characteristic equation of system; $K_{v}=K_{\text{пу}}\cdot K_{y}K_{\text{дв}}$ - coefficient of its amplification.

In accordance with expressions (4.5) and (4.6) transfer functions for the errors, caused by changes in angle ε and by drifts of gyroscope $\Delta\psi_{\text{г}}$, coincide. In the majority of the cases due to the smallness the velocities of the drifts/cares of the error of gyroscope can, apparently, be disregarded. Control signal of goniometer is removed/taken at point B of diagram, moreover

$$u_{\text{к}}=\frac{K_{\text{пу}}K_{y}}{N_{\text{к}}(D)}\dot{\varepsilon}+\frac{K_{\text{пу}}K_{y}}{N_{\text{к}}(D)}(\Delta\psi_{\text{г}}+\Delta\theta_{\text{п}}),$$

where

$$\dot{\varepsilon}=D\varepsilon,$$

together with useful component it contains the derivative of the

drifts of gyroscope $D\Delta\psi_{r}$ and fluctuation component, caused by the radio interference

$$\Delta\dot{\varepsilon}=\frac{K_{\text{пг}}K_{y}D}{N_{\text{к}}(D)}\Delta\theta_{\text{п}}.$$

The diagram of single-gyroscope coordinator examined is applicable only for the antenna systems, which have low overall sizes and weights. During the coupling relative to heavy antennas with the gyroscopic drive the best results gives use/application of gyroscopic stabilized platforms [5].

The use of gyroscopic drives, increases a quantity of gyroscopes on board the rocket; therefore in the principle it is possible to use for purposes of the stabilization of antenna attitude gyroscopes, although here appear the difficulties with the selection of the place of their installation.

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S4-3. Stabilization systems with the gyroscopes, established/installed on the housing of object.

The angle of rotation of object ϑ relative to inertial space can be measured by the displacement gyroscopes, adjusted on its housing and be introduced as the signal of self-contained meter into the

servo coordinator. Gyroscopes must be established/installed near the antenna. Therefore are excluded the disagreements in the angular velocities of different parts of the object due to the deformation of its housing.

The system block diagram of stabilization with the displacement gyroscopes, established/installed on the housing of object, is given in Fig. 4.5. Here R_o - feedback potentiometer; R_r - potentiometer of gyroscope.

Goniometer consists of two servo systems. The measuring element of radio engineering system reacts to the input effect

$$\gamma = \varepsilon - \vartheta.$$

As the measuring element/cell of the second self-contained duct/contour is used the displacement gyroscope, which reacts to changes in the attitude of the axis of rocket, which appears not only as a result of the disturbances/perturbations operating on it, but also due to the action of the system of steering of rocket.

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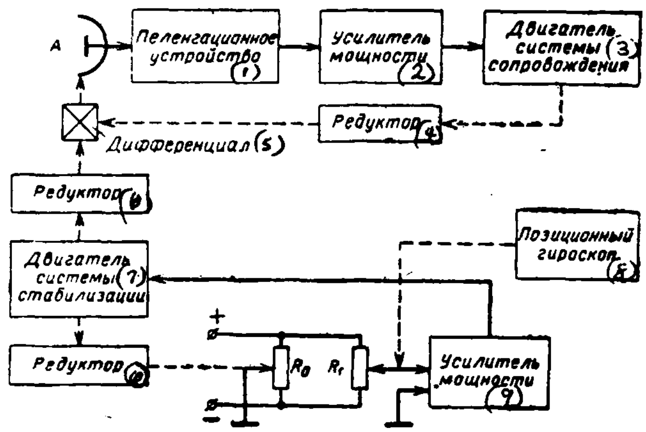

Fig. 4.5.

Key: (1). Direction-finding device/equipment. (2). Power amplifier. (3). Engine of tracking system. (4). Reducer. (5). Differential. (6). Reducer. (7). Engine of stabilization system. (8). Position gyroscope. (9). Power amplifier.

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With the misalignment of aircraft (rocket) relative to the direction, assigned by gyroscope, in potentiometric type bridge circuit appears the error signal, which after amplification enters the engine of stabilization system. The engine through the reducer and the differential turns the moving element of the antenna in the side, opposite to the misalignment of aircraft and simultaneously is

restored balance in the bridge circuit.

However, with the divergence of target from the equisignal direction, appears the error signal in the duct/contour of the tracking system of target. Under the effect of this signal the engine of tracking system through the same differential will turn antenna in the direction to the target.

It is natural that the rotation caused by control signal of the axis of rocket will be also measured by the displacement gyroscope and through the duct/contour of the stabilization it will turn antenna in the direction, which reduces the error signal at the output of direction-finding device/equipment. However, the inertness of control loop of rocket substantially exceeds the inertness of the ducts/contours of target tracking and the stabilization of antenna. Therefore the analysis of the dynamics of complex goniometer can be carried out without taking into account feedback through the control loop of rocket and to consider that the input of the duct/contour of stabilization enters the complete signal ϑ. It is necessary, however, to keep in mind that this analysis is valid at the small angles of rotation of the axis of rocket relative to its position at the moment of actuation of gyroscope. The block diagram of the system in question when making these assumptions can be represented in the form Fig. 4.6.

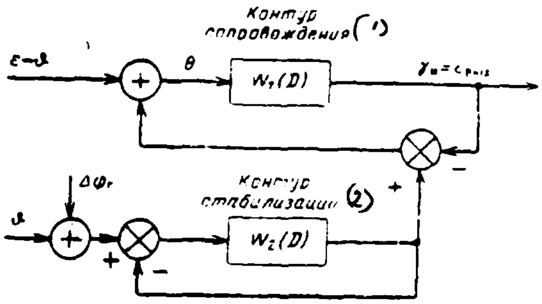

Fig. 4.6.

Key: (1). Tracking circuit. (2). Duct/contour of stabilization.

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As before, it relates to the class of systems with the position correction from the gyroscopic servo system (duct/contour of stabilization).

Signal $\gamma_н$ at the output of system without taking into account radio interferences is determined by the equation

$$\gamma_н = \Phi_1(D)\varepsilon + \Phi_1(D)[1 - \Phi_2(D)]\vartheta + \frac{1}{1+W(D)}\cdot\Phi_2(D)\,\Delta\psi_г, \qquad (4.7)$$

where $\Phi_1(D)$ - the transfer function of the locked duct/contour of target tracking; $\Phi_2(D)$ - the duct/contour of stabilization.

There is interest to consider expressions for the errors:

- in the accompaniments

$$\theta = (\varepsilon - \vartheta) - \gamma_{н}, \tag{4.8}$$

- the measurement the control signal

$$\Delta\varepsilon = \varepsilon - \gamma_{н}. \tag{4.9}$$

It is not difficult to show that

$$\begin{aligned} \theta &= [1 - \Phi_2(D)]\,\varepsilon - [1 - \Phi_1(D)] \times \\ &\times [1 - \Phi_2(D)]\,\vartheta - \frac{\Phi_2(D)}{1 + W_1(D)}\,\Delta\psi_{г}, \end{aligned} \tag{4.10}$$

and the component of the error of control signal, caused by the fluctuations of the axis of rocket and which characterizes the quality of the work of stabilization system,

$$\Delta\varepsilon_{\vartheta} = \Phi_1(D)\,[1 - \Phi_2(D)]\,\vartheta. \tag{4.11}$$

Formulas (4.10) and (4.11) show that the elimination of the effect of the angular motion of antenna mounting to the tracking errors and measurement of useful signal occurs both due to the functioning of stabilization system and due to the action of tracking system.

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It is natural that the parameters of tracking circuit the

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accuracy of accompaniment and measurement ε affect differently. The condition of the invariance of system relative to the fluctuations of the axis of rocket is the fulfillment of the equality

$$\Phi_2(j\omega)=1 \qquad (4.12)$$

in entire frequency spectrum of the angle ϑ.

It is natural that this condition can be satisfied only approximately; degree of approximation to condition (4.12) characterizes the quality of the work of stabilization system.

Transfer functions for all components of the error of output signal differ from each other; therefore there is a freedom in the selection of the optimum parameters, which minimize the error of reproduction.

If system did not have a duct/contour of the stabilizations [$W_2(D)=0$], transfer functions for both component signals ϑ and ε, which must track goniometer, they would coincide and the possibilities of the optimization of the parameters of system would be substantially limited. The decrease of the dynamic error of the tracking of angle ϑ due to the expansion of the passband of the duct/contour of stabilization is limited only to the design considerations: by the power of engines, with the accuracy of sensors

finally by the need for ensuring the stability of the duct/contour of stabilization, but it does not affect the error, caused by radio interferences.

For the rough estimate of the accuracy of system and effect on it of the parameters of ducts/contours let us assume that

$$W_1(D)=\frac{K_{v1}}{D};\quad W_2(D)=\frac{K_{v2}}{D}.$$

Under the done assumption of equation for the errors, caused by the fluctuations of the axis of object θ_ϑ and $\Delta\varepsilon_\vartheta$ it will take the form:

$$\theta_\vartheta=\frac{D^2\vartheta}{D^2+(K_{v1}+K_{v2})D+K_{v1}K_{v2}},\qquad (4.13)$$

$$\Delta\varepsilon_\vartheta=\frac{DK_{v1}\vartheta}{D^2+(K_{v1}+K_{v2})D+K_{v1}K_{v2}}.\qquad (4.14)$$

Let us compute dispersions of both component errors on the assumption that the fluctuations of the axis of rocket have random character and are assigned by the spectral density

$$S_\vartheta(\omega)=\frac{2\beta\sigma_\vartheta^2}{\omega^2+\beta^2}.\qquad (4.15)$$

where σ_ϑ^2 - dispersion of angle ϑ.

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After using formula (3.37), we will obtain expressions for the

variances of error

$$\sigma_{\theta}^{2}=\frac{\sigma_{\phi}^{2}\beta\,[(1+a_{v})\beta+K_{v1}a_{v}]}{[K_{v1}(1+a_{v})^{2}\beta+(1+a_{v})\beta^{2}+K_{v1}(a_{v}+1)a_{v}]},\quad(4.16)$$

$$\sigma_{\varepsilon}^{2}=\frac{\sigma_{\phi}^{2}K_{v1}\beta}{[K_{v1}(1+a_{v})^{2}\beta+(1+a_{v})\beta^{2}+K_{v1}(a_{v}+1)a_{v}]},\quad(4.17)$$

where $a_v=K_{v2}/K_{v1}$ - ratio of the factors of amplification of the ducts/contours of stabilization and tracking.

Variances of error for the nonstabilized system can be obtained from (4.16) and (4.17), if we in them assume K_{v2}, and respectively a_v equal to zero, i.e.,

$$\sigma_{\theta н}^{2}=\frac{\sigma_{\phi}^{2}\beta}{K_{v1}+\beta},\quad(4.18)$$

$$\sigma_{\varepsilon н}^{2}=\frac{\sigma_{\phi}^{2}K_{v1}}{K_{v1}+\beta}.\quad(4.19)$$

Comparing formulas (4.18) and (4.19), it is not difficult to see that with an increase in the passband (by increase K_{v1}) $\sigma_{\theta н}$ it vanishes, and $\sigma_{\varepsilon н}$ it approaches σ_{ϕ}, which confirms the observation expressed above about the discrepancy of requirements about the accuracy of accompaniment and accuracy of reproduction of control signal ε.

The contradiction indicated to a considerable degree is permitted by the inclusion into the goniometer of the duct/contour of stabilization.

Gain in the accuracy of the tracking of input signal due to the

introduction of the duct/contour of stabilization is convenient to determine in the form of the ratio

$$b_{\beta} = \frac{\sigma^2_{\beta M}}{\sigma^2_{\beta}} = \frac{K_{v1}(1+a_v)^2\beta + (1+a_v)\beta + K^2_{v1}(a^2_v + a_v)}{(K_{v1}+\beta)\left[(1+a_v)\beta + a_v K_{v1}\right]}. \quad (4.20)$$

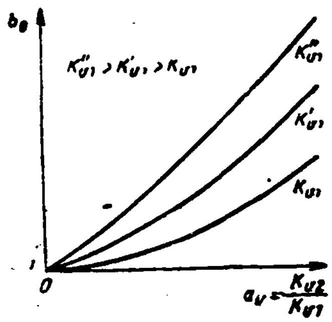

Fig. 4.7.

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The character of dependence b_θ on a_v at different values K_{v1} is shown on the graphs Fig. 4.7, from which it is evident that the gain the higher, is the more K_{v1} and a_v.

The introduction of the duct/contour of stabilization raises to the accuracy of reproduction of signal $\dot{\varepsilon}$. Gain is conveniently characterized by the relation

$$b_\varepsilon = \frac{\sigma^2_{\varepsilon 0}}{\sigma^2_\varepsilon} = \frac{K_{v1}(1+a_v)\beta + (1+a_v)\beta^2 + K_{v1}(a_v^2 + a_v)}{(K_{v1}+\beta)\beta}. \quad (4.21)$$

In this case the gain turns out to be even greater than in terms of the accuracy of tracking the input effect.

The introduction of the duct/contour of stabilization causes the appearance one additional component of the measuring error and

accompaniments, caused by the gyroscope drift.

Equation for this error (4.10) takes the form

$$\theta_{\psi} = \frac{DK_{v2}\Delta\psi_{r}}{D^{2} + (K_{v1} + K_{v2})D + K_{v1}K_{v2}}. \qquad (4.22)$$

The error of gyroscope approximates well expression (1.43).

The velocity of the drift of gyroscope b_1 and random drift $\xi_r(t)$ in the dependence on the type of gyroscope can vary in the very wide limits. But the component of random drift, as a rule, in the stabilization systems can be disregarded/neglected. The steady tracking error, caused by the "regular" gyroscope drift, is determined by the formula

$$\theta_{\psi 0} = \frac{b_1}{K_{v1}}. \qquad (4.23)$$

Since b_1 for the serial gyroscopes composes the value, which does not exceed 1 deg/h, then by the measuring error of angle ε, that caused by this drift/care, can be virtually disregarded/neglected.

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For obtaining the signals, proportional to the rate of the motion of equisignal direction $\dot{\varepsilon}$, on the axis of the engines of

final adjustment system are established/installed the tachometers. Equations for the output signals and the errors of system with the tachometers can be obtained by the multiplication of the right sides of equations (4.10) and (4.11) to $K_{\tau\tau}D$, where $K_{\tau\tau}$— the transmission factor of tachometer. Respectively it is changed and the filtering properties diagrams of the disturbances/perturbations relatively operating on it. One should again note that due to divergence of gyroscope and antenna the efficiency of stabilization system falls from an increase in the angle γ.

To merits of the system examined should be related the possibility to virtually completely shift the problem of the tracking of the angular motion of rocket to the duct/contour of the stabilization. However, system is bulky, it must have two actuators, adders in the form of differential for each plane, etc.

More simply is realized diagram with one servomotor (Fig. 4.8).

As the meter of the angular oscillations of rocket, as in the preceding case, is used the displacement gyroscope.

Since servomotor is the integrating element/cell, for agreeing the dimensionalities of basis and corrective commands it is encompassed by rigid negative feedback. Because of the presence of

this connection/communication is formed the component/link with the transmission factor, equal to one at the frequency $\omega=0$, equivalent to inertial, oscillatory or even more complicated dynamic component/link. For achievement of astaticism into the duct/contour of the target tracking is switched on further integrator (usually subminiature motor).

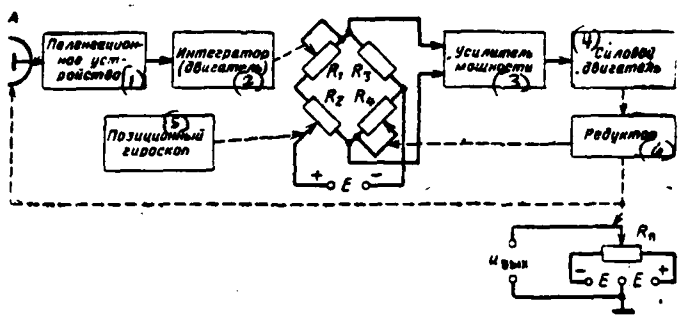

Fig. 4.8.

Key: (1). Direction-finding device/equipment. (2). Integrator (engine). (3). Power amplifier. (4). Power engine. (5). Displacement gyroscope. (6). Reducer.

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The signal, which appeared with a change in angle ε, at the output of direction-finding device/equipment causes the rotation of the integrating engine which by the rotation of the wiper of potentiometer R_1 causes the imbalance of bridge, the appearance of voltage/stress in its diagonal and the rotation of the servomotor which will turn antenna and, after shifting the wiper of potentiometer R_4, it will restore/reduce balancing/trimming bridge. Antenna will swing through angle $\varepsilon_{вых}$. The circuits examined compose the tracking circuit of target. If is deflected the axis of the

rocket (will be changed angle ϑ), then the axis of gyroscope will remain on the spot, and the wiper of potentiometer R_2, connected with it, attached on the missile body, will change its position, unbalances bridge, will appear voltage on the input of power amplifier and antenna will begin to move in the direction, opposite to the misalignment of rocket. So works the duct/contour of the stabilization. The required phasing of system is reached by the selection of the connection point of potentiometer R_2 and direction of rotation of the wipers of potentiometers R_1 and R_4.

Between the yaw angles and the error signal at the output of bridge circuit is a nonlinear dependence, but at the small displacement angles diagram it is possible to linearize. Then the structural schematic of system will take the form, depicted in Fig. 4.9 [1].

FOOTNOTE [1]. In the practical diagrams frequently use the adders in the form of differential selsyns [14]. Bridge circuit is given here as the simplest version, convenient for the explanation of operating principle. ENDFOOTNOTE.

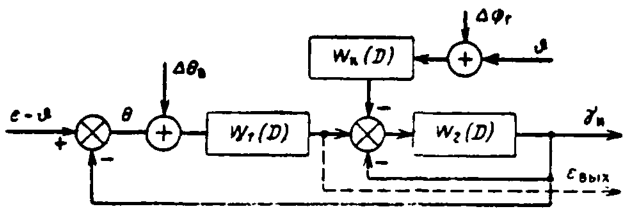

Fig. 4.9.

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Here:

$W_1(D)$ - the transfer function of direction-finding device/equipment and drive of the wiper;

$W_2(D)$ - the transfer function of actuating element;

$W_к(D)$ - transfer function of correcting term whose inclusion in certain cases can prove to be appropriate.

The diagram in question also gives the possibility to simultaneously ensure the high quality of the accompaniment of signal

γ and to exclude from control signal $\varepsilon_{вых}$ component, caused by the fluctuations of the axis of rocket. According to the classification given above it relates to the class of systems with the position correction.

The transfer function of servomotor with the power amplifier, included by negative feedback, let us designate through

$$\Phi(D)=\frac{W_2(D)}{1+W_2(D)}. \qquad (4.24)$$

Output signal in the absence of radio interferences $(\Delta\theta_п=0)$ is described by the expression

$$\gamma_в=\frac{W_1(D)\Phi(D)}{N_2(D)}\varepsilon-\frac{W_к(D)\Phi(D)+W_1(D)\Phi(D)}{N_2(D)}\vartheta-$$
$$-\frac{W_к(D)\Phi(D)}{N_2(D)}\Delta\psi_г, \qquad (4.25)$$

where

$$N_2(D)=1+W_1(D)\Phi(D);$$

$\Delta\psi_г$ - the error of gyroscope.

Equation for the error signal takes the form

$$\theta=\varepsilon-\vartheta-\gamma_в=\frac{1}{N_2(D)}\varepsilon-\frac{\Phi(D)W_к(D)}{N_2(D)}\Delta\psi_г-$$
$$-\frac{[1-\Phi(D)W_к(D)]}{N_2(D)}\vartheta. \qquad (4.26)$$

From the obtained expression it follows that there is at least

two possibilities to do invariant system to the signal ϑ. The first path consists of the creation of the servomechanism (internal control loop) with the large passband.

In fact, after placing $W_{к}(D)=1$, the condition of invariance it is possible to obtain in the form

$$\Phi(j\omega)=1$$

in the entire band of frequencies of the angle ϑ.

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There is another path: by correspondingly to form corrective command due to the selection of the transfer function of correcting term.

The condition of invariance it will satisfy correcting term, which has transfer function (4.24):

$$W_{к}(D)=\frac{1}{\Phi(D)}=\frac{1+W_{2}(D)}{W_{1}(D)}. \qquad (4.27)$$

Transfer function of the engine

$$W_{2}(D)=\frac{K_{дв}}{D(T_{дв}D+1)}.$$

Then the desired value

$$W_{\kappa}(D)=\frac{T_{2\text{в}}D^{2}+D^{2}+K_{2\text{в}}}{K_{2\text{в}}}.$$

Even the approximate realization of this component/link presents considerable difficulties. But for this, as a rule, and there is no need.

If we exclude corrective unit [which corresponds to replacement in (4.26) (4.27) $W_{\kappa}(D)$ per unit] and to disregard/neglect the inertness of direction-finding device/equipment, i.e., to place $W_{1}(D)=K_{\text{пу}}/D$. the equation for the component of the following error, caused by changes in the angle ϑ, will take the form

$$\theta_{\vartheta}=\frac{D(T_{2\text{в}}D+1)\,\vartheta}{T_{2\text{в}}D^{3}+D^{2}+K_{2\text{в}}D+K_{v}}. \tag{4.28}$$

When $K_{v}=K_{\text{пу}}K_{\text{дв}}\rightarrow\infty,\ \theta_{\vartheta}\rightarrow 0.$

The error of reproduction ε is determined by the equation

$$\theta_{\varepsilon}=\frac{D(D+K_{2\text{в}})\,\varepsilon}{T_{2\text{в}}D^{3}+D^{2}+K_{2\text{в}}D+K_{v}}. \tag{4.29}$$

Whence conservative value of velocity error

$$\theta_{\dot{\varepsilon}}=\frac{\dot{\varepsilon}}{K_{\text{пу}}} \tag{4.30}$$

As one would expect, it does not depend on the transmission factor of

engine $K_{дв}$.

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Equation for the component of the error, caused by the gyroscope drift, takes the form

$$\theta_{\psi} = \frac{(K_{дв} + T_{дв}D)D}{T_{дв}D^3 + D^2 + K_{дв}D + K_{\psi}} \cdot \Delta\psi_{г},\qquad (4.31)$$

but velocity error is determined by the formula

$$\theta_{\dot{\psi}} = \frac{b_1'}{K_{дв}}.\qquad (4.32)$$

Thus from the point of view of the minimization of the error of the tracking of input signal system with the general/common/total for the circuits of accompaniment and stabilization by the actuator possesses the same possibilities as the examined in previous paragraph system with the separate drives. However, to use as control signal $\gamma_{н}$ is inexpedient, since it contains signal ϑ.

In fact, the component of output signal (4.25), caused by angle ϑ,

$$\gamma_{н\vartheta} = -\frac{W_{н}(D)\Phi(D) + W_1(D)\Phi(D)}{1 + W_1(D)\Phi(D)}\vartheta.\qquad (4.33)$$

As already mentioned, to minimize tracking error is possible with

$$|\Phi(j\omega)| \approx 1.$$

Then the component of control signal, caused by angle ϑ, is equal to

$$\gamma_{\text{н}\vartheta} \approx \frac{1+W_1(D)}{1+W_1(D)}\vartheta = \vartheta. \qquad (4.34)$$

In contrast to the diagram in Fig. 4.5 output shaft of the engine with which is connected the wiper of measuring potentiometer $R_{\text{п}}$, it obtains motion both with the change ε, and with the change ϑ.

However, the deficiency/lack indicated is easily reduced. It suffices control signal $\varepsilon_{\text{вых}}$ to remove/take from the output of the integrator (see Fig. 4.9).

The component of the error of control signal $\varepsilon_{\text{вых}}$, caused by the fluctuations of the axis of rocket,

$$\Delta\varepsilon_{\varepsilon\,\text{вых}} = \frac{W_1(D)[1-\Phi(D)]}{1+W_1(D)\Phi(D)}\vartheta \qquad (4.35)$$

when $\Phi(j\omega)=1$ becomes zero.

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Thus, diagrams in Fig. 4.5 and 4.8 are equivalent on the dynamic ones by properties, but the second proves to be more simply. While conducting of the analysis of the dynamic properties of complex goniometers it was assumed that the rocket does not have banks. Only under this condition the channels of measurement and stabilization in both planes prove to be independent variables. If for the rockets, although approximately, this is admissible, then for the aircraft it is erroneous, since the speeds of banks can several times exceed the yaw rates and pitch.

To eliminate the effect of banks it is possible in two ways:

- antihunting antenna along the bank;

- to carry out translation of the stabilizing signals taking into account the measured roll attitude, using the special computer. In detail this question is illuminated in [14]. Are there given equations of relation of the signals of stabilization for all three channels.

§4.4. Stabilization system of antennas with the gyroscopes, established/installed on the moving element of the antenna.

Together with coordinators examined above with the gyroscopes, adjusted on the housing of object, are used the systems with the gyroscopic sensors, adjusted on the moving element of the antenna, in particular on reflector itself [14, 11]. This provides the measurement of position or speed of the displacement/movement of antenna in the azimuthal plane and the planes of angle of elevation.

Let us consider the stabilization system in which as the measuring element/cell is used the displacement gyroscope, established/installed on the moving element of the antenna. The output signal of this gyroscope in the absence of interferences will consist of two components:

- caused by displacement/movement antenna, caused by the drive of tracking circuit;

- caused by the angular oscillations of rocket.

If tracking circuit worked without the error, then second component would be equal to zero, since in this case tracking circuit would completely parry the angular oscillations of rocket.

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This cannot be attained due to the limitedness of the band of tracking circuit. Moreover, this should not be attained, since the expansion of the passband of coordinator leads to an increase in the errors, caused by the fluctuations of the echo signal and by radio interferences. The partial decoupling of the angular oscillations of rocket in the coordinators of the type in question is reached because in the output signal of gyroscope (with the limited band of servo system) is contained the information about the angular oscillations of the axis of rocket.

The functional diagram of stabilization system with the use as sensing element of displacement gyroscope [11] is given in Fig. 4.10. Signal from the output of direction-finding device/equipment in the system in question is supplied on the circuit of the correction of gyroscope and is oriented its axis in the process of moving the rocket along the equisignal direction. During the fluctuations of the axis of rocket (changes in the angle ϑ) appears the signal both at the output of direction-finding device/equipment and at the output of gyroscope. Signal from the output of gyroscope enters the engine of tracking system, included by high-speed/high-velocity feedback.

Let us construct the block diagram of system. With respect to

the signal, which corrects the position of sensitive axis (i.e. in the tracking circuit), the gyroscope behaves as the integrating component/link, since this signal is supplied to the torque motor, which calls precession. In the duct/contour of the stabilization of antenna the gyroscope fulfills other functions.

With a change in the position of the axis of rocket to the angle ϑ at the output of gyroscope appears further signal - ϑ_{r}, since the axis of gyroscope retains its attitude sensing. Since, however, the gyroscope is established/installed on the antenna, it will master additionally angle γ_{m}, to which it is turned antenna.

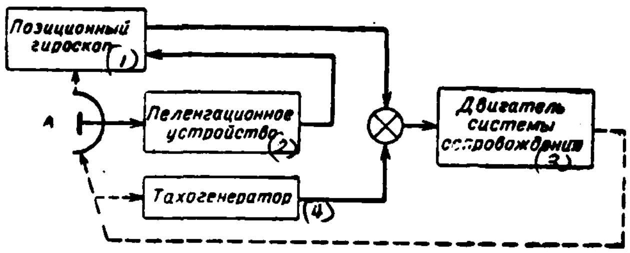

Fig. 4.10.

Key: (1). The displacement gyroscope. (2). Direction-finding device/equipment. (3). Engine of tracking system. (4). Tachogenerator.

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Consequently, the common output angle of the gyroscope

$$\gamma_{г}=\gamma_{пг}+(-\vartheta-\gamma_{п}+\Delta\varphi_{г}), \qquad (4.36)$$

where $\gamma_{пг}$ - precession angle, caused by the signal of direction-finding device/equipment. (Latter it is caused by disagreement/mismatch θ).

With precision work of system the angle between the sensitive axis of gyroscope and potentiometer $\gamma_{г}=0$.

Thus, was delineated the block diagram of system (Fig. 4.11), where it is marked:

$W_1(D)$ - transfer function of direction-finding device/equipment and circuits of the correction of gyroscope;

$W_2(D)$ - transfer function of the engine of tracking system;

$W_3(D)$ - transfer function of gyroscope with respect to the corrective voltage/stress;

$W_4(D)=K_{\text{тг}}D$ -the transfer function of tachogenerator.

The negative inverse tachometer connection/communication, used in the described diagram, makes it possible to reduce the inertness of engine. It is easy to show that the equivalent time constant of the engine

$$\tau_э=\frac{T_{дв}}{K_{\text{тг}}K_{дв}+1}.$$

Choosing $K_{\text{тг}}$ large, it is possible to strongly decrease the inertness of engine, true, due to the decrease of transmission factor. One ought not, of course, to think that the tachogenerator is

applicable only in this diagram.

Comparing diagrams in Fig. 4.11 and 4.9, it is possible to ascertain that they are equivalent. Moreover, from the point of view of errors nothing it will be changed, if gyroscope with the slide/wiper is established/installed on the housing of object, but the housing of potentiometer - on the moving element of the antenna.

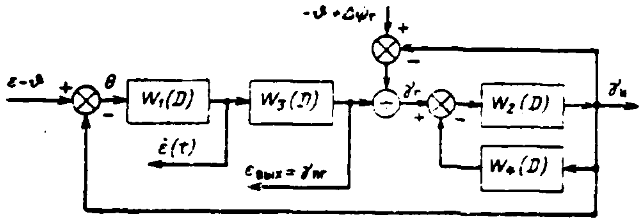

Fig. 4.11.

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Let us note that the constant errors of gyroscope are localized in all systems examined. Physically this is explained by the fact that the system is static and if at the initial moment with t=0 gyro output signal is not equal to zero, then appears output potential of direction finder, as a result of which the wiper will be displaced to the angle at which the output voltage/stress of gyroscope will become equal to zero.

It is important to note that the errors of gyroscope, caused by friction in the suspension and imbalance, that are transient in the extended gyroscope (see Chapter 1), they do not produce the transient errors of servo system. Of this it is not difficult to be convinced, after leading error $\Delta\psi_r$ to the input of gyroscope.

It is possible to use the system in question, also, for the measurement by the derivative of angle $\varepsilon(t)$. Signal at the output of direction finder as this is shown in Fig. 4.11, is proportional $\dot{\varepsilon}(t)$ However, where it is necessary to measure the signal, proportional $\dot{\varepsilon}(t)$, prefer the diagram, analogous to that examined, but with the rate gyroscope as the measuring element of stabilization system.

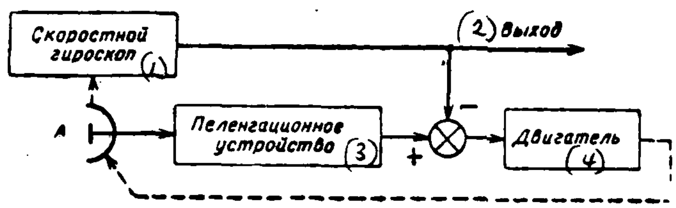

Fig. 4.12.

Key: (1). The rate gyroscope. (2). output. (3). Direction-finding device/equipment. (4). Engine.

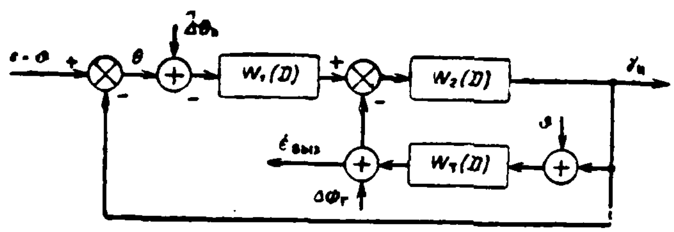

Fig. 4.13.

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The output signal of gyroscope can be introduced into the tracking circuit of target as this shown in Fig. 4.12, and thereby to parry the large part of the angular displacements of rocket, after

facilitating working conditions of tracking circuit.

The block diagram of this system is given in Fig. 4.13. Here

$W_1(D)$ - the transfer function of direction finder;

$W_2(D)$ - the transfer function of executive drive;

$W_3(D)$ - the transfer function of the rate gyroscope.

The gyroscope, established/installed on the moving element of the antenna, measures the angular velocity of its motion, since the axis of gyroscope is turned to the lead angle relative to the axis of rocket and is measured, as in the previously diagrams examined, only the projection of angular oscillations on the equisignal direction. This fact is very undesirable, since stabilization system proves to be nonlinear, and the quality of its work will vary depending on lead angle. Certainly, having a signal, proportional to mentioned angle (γ_H in Fig. 4.13), it is possible to carry out translation of the measured signal before its supply to the input of executive drive. The characteristic feature of the diagram in question is that to obtain the signal, proportional to the orientation angle of antenna in fixed coordinate system

$$\varepsilon_{\text{вых}}(t)=\int_0^t \dot{\varepsilon}(\tau)\,d\tau, \qquad (4.37)$$

is possible only due to the integration of signal $\dot{\varepsilon}(\tau)$ by separate integrator. Therefore coordinators of such type are used only, where for the rocket control (aircraft) it suffices to have a signal the proportional $\dot{\varepsilon}$, which is removed/taken either from the output of the rate gyroscope or from the output of direction-finding device/equipment.

On the system operate the fluctuations of the signal echo from the target and radio interference, considered by angle $\Delta\theta_{п}$. The errors of gyroscope are taken into consideration by introduction to the system of signal $\Delta\psi_{г}$.

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It is easy to show that the control signal

$$\dot{\varepsilon}_{\text{вых}} = \frac{W_1(D)\,W_2(D)\,W_3(D)}{N_c(D)}(\varepsilon+\Delta\theta_{п}) + \frac{W_3(D)}{N_c(D)}\Delta\psi_{г} + \frac{W_2(D)\,W_3(D)}{N_c(D)}\,\vartheta, \qquad (4.38)$$

where

$$N_c(D)=1+W_1(D)\,W_2(D)+W_2(D)\,W_3(D). \qquad (4.39)$$

Satisfactory results from the point of view of the errors of

control signal $\Delta\dot{\varepsilon}$ and accompaniment θ it is possible to obtain, assuming that consecutively/serially with the engine of tracking system is connected the further integrator. The recommendations of this character are contained also in [14]. For simplification in the analysis let us assume that $W_2 = K_{\text{и}} K_{\text{дв}}/D^2$, $W_1 = K_{\text{пу}}$. Analysis shows that the condition of the absence of static errors requires placing in the transfer function of rate gyroscope $W_3(D)=K_3D$, coefficient $K_3=1$ s/rad.

For obtaining the equation of error let us replace

$$W_3(D)\varepsilon(t) = D\varepsilon(t) = \dot{\varepsilon}(t)$$

Then equation (4.38) will take the form

$$\Delta\dot{\varepsilon} = \dot{\varepsilon} - \dot{\varepsilon}_{\text{вых}} = \frac{D(D + K_{\text{и}}K_{\text{дв}})}{F(D)}\dot{\varepsilon} - \\ - \frac{K_v}{F(D)}\Delta\theta_{\text{п}} - \frac{D^2}{F(D)}\vartheta + \frac{DK_{\text{и}}K_{\text{дв}}}{F(D)}\Delta\psi_{\text{г}}, \qquad (4.40)$$

where $F(D) = D^2 + K_{\text{и}}K_{\text{дв}}D + K_v$ — the characteristic equation of system; $K_v = K_{\text{и}}K_{\text{дв}}K_{\text{пу}}$ - factor of amplification of tracking system in the speed.

Conservative values of the errors, caused by input effect ε and drift of gyroscope $\Delta\psi_{\text{г}}$, have values

$$\Delta\dot{\varepsilon}_{\varepsilon} = \frac{\ddot{\varepsilon}}{K_{\text{пу}}}, \quad \Delta\varepsilon_{\psi} = \frac{b_1}{K_{\text{пу}}}, \qquad (4.41)$$

where b_1 - speed of the gyroscope drift.

Formula (4.41) indicates which to minimize errors is possible only due to an increase in the transmission factor of direction-finding device/equipment. This result it is easy to explain.

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Final adjustment system, which is series connection of two integrators, included by high-speed/high-velocity negative feedback has the transfer function

$$\Phi_c(D)=\frac{1}{D\left(\frac{1}{K_{дв}}D+1\right)}, \qquad (4.42)$$

i.e. it is equivalent to series connection of the integrator, which has the transmission factor, equal to one, and inertial component/link.

If with the engine was not consistently connected further integrator, then goniometer would prove to be static system. In fact, after assuming $W_2(D)=K_{дв}/D$, we will obtain

$$\Phi'_c(D)=\frac{1}{\frac{1}{K_{\text{зв}}}D+1}. \qquad (4.43)$$

Equation (4.40) shows that the system stresses the high-frequency components of signal ϑ, therefore, possibly, is required the further smoothing of control signal.

Following error is described by the expression

$$\theta=\varepsilon-\vartheta-\gamma_{\text{н}}=\frac{D(D+K_{\text{зв}})}{F(D)}\varepsilon-\frac{D^2}{F(D)}\vartheta+$$
$$+\frac{K_{\text{зв}}D}{F(D)}\Delta\psi_{\text{г}}-\frac{K_vD}{F(D)}\Delta\theta_{\text{п}}. \qquad (4.44)$$

With respect to the drifts of gyroscope $\Delta\psi_{\text{г}}$ the system is astatic. Conservative value of velocity error is equal

$$\theta_v=\frac{b_1}{K_{\text{пу}}}$$

and practical limitations on selection $K_{\text{пу}}$ it does not place.

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S4.5. Systems of the "stabilization of data" and the combined systems.

All systems examined above provided the stabilization of antenna

position. In this case it was possible to facilitate working conditions of tracking circuit and to simultaneously decrease the error of control signal, caused by the fluctuations of the axis of rocket.

In the systems of the "stabilization of data" is solved only second problem. However, the accompaniment of target is realized by a usual, unintegrated goniometer. An example of a similar system is examined in work [25]. In the work indicated the examination is led in connection with phase type monopulse system, which has two pairs of antennas, rigidly attached on the missile body.

The tracking of target by an equisignal direction is realized due to phase compensation of signals in each of the pairs of the antenna-waveguide systems, and obtaining the signal, proportional to $\dot{\varepsilon}$, is realized due to the rotation of the rate gyroscope, established/installed on the housing of object by the system of the final adjustment of phase. In the same plan/layout, in which were examined all previous systems, it is possible to analyze the quality of the output signal of system irrespectively of the type of direction-finding device/equipment, including direction-finding device/equipment, which has mobile antenna.

With the mobile antenna the system of the "stabilization of

data" structurally/constructurally coincides with system examined above with the rate gyroscope, with that difference, that the signal of the rate gyroscope is not introduced into the tracking circuit.

Fig. 4.14 gives the simplified functional diagram of system, while in Fig. 4.15 - its block diagram.

As in the previously systems examined, the role of the differentiator, necessary for obtaining of signal , performs the rate gyroscope, which reacts both to the rotation of the axis of coordinator and to the fluctuations of the axis of rocket.

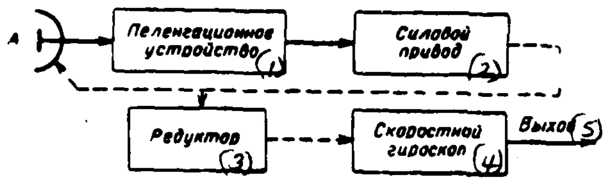

Fig. 4.14.

Key: (1). Direction-finding device/equipment. (2). Actuator. (3). Reducer. (4). Rate gyroscope. (5). output.

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The "stabilization" of data is provided due to the subtraction from the output signal of the servo system of the signal, proportional ϑ.

The output signal of complex meter will be determined by the equation

$$\varepsilon_{вых} = \Phi_д(D) K_{1ед} K_з D\varepsilon + \\ + K_з(D)\left[1 - K_{1ед}\Phi_д(D)\right]\vartheta + \Delta\psi_г, \qquad (4.45)$$

where

$$\Phi_д(D) = \frac{W_1(D)\,W_2(D)}{1 + W_1(D)\,W_2(D)}$$

- transfer function of closed system;

$K_{ред}$ - transmission factor of the reducer, included between the rate gyroscope and the system of final adjustment with the fixed antenna;

K,D - transfer function of the rate gyroscope.

Parameter $\dot{\varepsilon}_{вых}$ serves for changing the velocity vector of rocket in such a way that $\dot{\varepsilon}$ it would vanish.

At the same time from the examination of expression (4.45) it is evident that the selection of the transmission factor of reducer $K_{ред}$ it is possible to make a coefficient when ϑ both positive and negative.

In the phase monopulse caps, as shown in work [25], this sets substantial limitations on entire dynamics of homing systems.

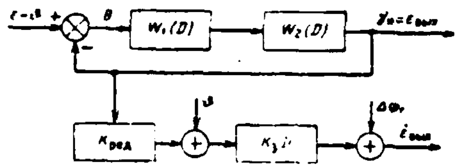

Fig. 4.15.

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After placing in expression (4.45) $W_1 = K_{пу}$, $W_2(D) = K_{дв}/D$, equation for the output signal we will obtain in the form

$$\dot{\varepsilon}_{вых} = \frac{K_з K_{рел} K_v \dot{\varepsilon}}{D + K_v} + \frac{K_з K_{рел}(K_v + D - K_v K_{рел})}{D + K_v}\dot{\vartheta} + \Delta\psi_{r_t} \qquad (4.46)$$

where $K_v = K_{пу} K_{дв}$ - factor of amplification of system in the speed.

From the examination of expressions (4.46) it is evident that compensation condition in the output signal of component, proportional $\dot{\vartheta}$, requires selection $K_{рел} = 1$. in this case the output signal

$$\dot{\varepsilon}_{вых} = \frac{K_з \dot{\varepsilon}}{\frac{1}{K_v} D + 1} + \frac{\frac{K_з}{K_v}\ddot{\vartheta}}{\frac{1}{K_v} D + 1} + \Delta\psi_r \qquad (4.47)$$

contains the error, proportional to the second derivative of signal ϑ.

Together with the systems, which realize the principle of the stabilization of antenna or the "stabilization of data", they can find use and combined systems, which use both principles indicated simultaneously.

Thus, the complex goniometer whose block diagram is shown in Fig. 4.9, can be converted to the form, depicted in Fig. 4.16.

Control signal here is obtained as a result of subtraction from signal $\gamma_{\text{н}}$ of the signal of gyroscope. With the ideal tracking of the input effect, caused by the angular oscillations of the axis of object (angle ϑ), it will be completely excluded from the control signal as in the diagram in Fig. 4.9. From the point of view of the errors of those caused $\Delta\psi_r$ and $\Delta\theta_{\text{н}}$ the diagram in Fig. 4.16 also is virtually equivalent to diagram in Fig. 4.9. Nevertheless the method of the combined use of signals of gyroscope in certain cases can prove to be useful.

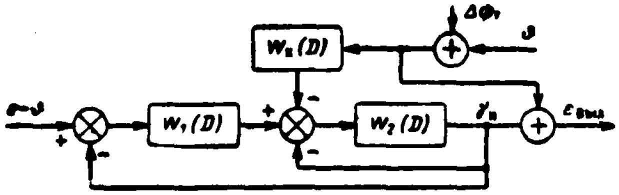

Fig. 4.16.

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Chapter 5.

INTEGRATED AUTOMATIC RANGING SYSTEMS.

The radio engineering servo range finder, established/installed on the mobile foundation (for example, flight vehicle) and which measures the distance of any other object (target or beacon) can be united into the integrated system by using the signals ISD. As such meters can be used the sensors of airspeed or accelerometers with the integrators. In both cases at the output is obtained the value, proportional to the speed of flight vehicle v_a, measured with some errors φ_a (see S1.2 and 1.4).

In this chapter are examined the characteristics of the complex range finders where the automatic determination of distance of the mobile object is based on the measurement of the temporary/time time lag of the impulses/momenta/pulses, reflected from these objects, i.e., is used pulse range-only radar with astaticism of the 1st or

2nd order. In this examination are not considered the fluctuations of the amplitudes of the signals echo from the target, which considerably facilitates conducting analysis. The given graphs of the errors, obtained according to precise formulas (see S3.4-3.6) and approximatio formulas of this chapter, make it possible to rate/estimate clearly the errors, caused by the assumption indicated.

S5.1. Block diagrams of complex range finders.

The functional diagram of range-only radar was described into S1.1.

In the practice are used the range-only radar with astaticism of the 1st and 2nd orders, which contain respectively one or two integrators in the control system.

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The signal of correction from ISD, proportional to th speed of displacement/movement v_a of flight vehicle, must be introduced through the appropriate filter into the point of range-only radar where occurs the signal, proportional to rate of change in the measured distance.

During the construction of the block diagram of range-only radar let us take into account that during the noise effect at the input of radio receiver the temporary/time discriminator in accordance with what has been said into S1.1 can be represented for the low disagreements/mismatches in the form of proportional dynamic component/link with the random transmission factor $K_{\text{вр}}(t)$ and by additive noise interference $f_{\text{р}}(t)$. The detailed proof of this position is given in the application/appendix.

Taking into account the aforesaid the block diagrams of complex range finder with by one and two integrators can be represented as in Fig. 5.1 and 5.2.

On the input of range finder acts the useful signal

$$x_{\text{д}}(t)=Д(t)+Д_{\text{а}}(t),$$

where $Д_{\text{а}}(t)$ - signal informatio about which can be obtained also from the self-contained meter (i.e. the component of range, which considers the motion of flight vehicle);

D(t) - signal information about which cannot be obtained from the self-contained meter (i.e. the component of range, which considers the displacement/movement of target).

Let us note that here and subsequently under the components of

range $Д_a(t)$ and D(t) are understood the projections of corresponding components on the direction flight vehicle - target.

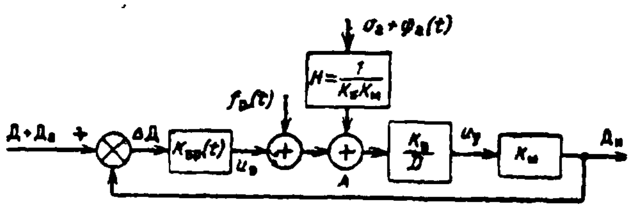

Fig. 5.1.

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The signal of the speed of self-contained meter v_a and the error of this meter through the coordinating filter with the transmission factor $H=\frac{1}{K_и K_м}$ for the system Fig. 5.1 and $H=\frac{1}{K_{и1}K_{и2}}$ - Fig. 5.2 - is introduced into point A of system. Through $K_и$, $K_{и1}$, $K_{и2}$ are designated the transmission factors of integrators, $K_м$ - coefficient, which characterizes the dependence of the temporary displacement of the strobe pulses converted in changes in the range, from voltage/stress u_y on the input of the device/equipment of the time delay:

$$K_м=\frac{dД_з}{du_y}\Big|_{u_y=u_{y0}}=\frac{dt_з}{du_y}\Big|_{u_y=u_{y0}}\cdot\frac{c}{2}=K_y\frac{c}{2}\left(\frac{м}{в}\right).$$

Here c - velocity of propagation of radio waves;

$t_з$ - temporary/time displacement of the strobe pulses of that of relatively sounding,

$Д_{и}(t)$ - the measured target range;

$$K_{\gamma}=\frac{dt_{и}}{du_{\gamma}}\Big|_{u_{\gamma}=u_{\gamma 0}}\left(\frac{сек}{в}\right).$$

[сек = s]

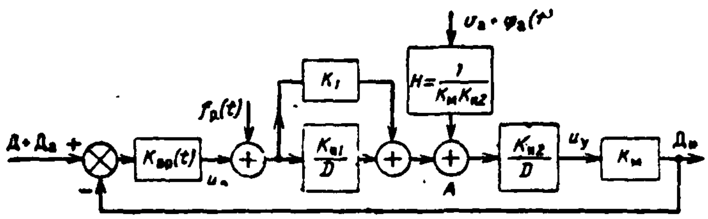

Fig. 5.2.

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If we suppose that $K_{вр}(t)$ - constant value equal to $K_{вр0}$, i.e. to disregard/neglect the fluctuations of the amplitudes of the echo signals then for the complex range finder with one integrator we will obtain

$$Д_и = Д_з + \frac{K_0}{D+K_0} Д + \frac{K_и K_м}{D+K_0} f_р + \frac{1}{D+K_0} \varphi_з, \quad (5.1)$$

but with two integrators

$$Д_и = Д_з + \frac{K_п D + K_a}{D^2 + K_п D + K_a} Д + \frac{K_п D + K_{и1} K_{и2} K_м}{D^2 + K_п D + K_a} f_р +$$
$$+ \frac{D}{D^2 K_п D + K_a} \varphi_з(t), \quad (5.2)$$

where $K_0 = K_{вр0} K_и K_м$ - transmission factor on the speed;

$K_a = K_{вр0} K_{и1} K_{и2} K_м$ - transmission factor on the acceleration:

$$K_п = K_{вр0} K_1 K_{и2} K_м; \quad (5.3)$$
$$K_и = K_1 K_{и2} K_м.$$

Respectively the error of the reproduction of the system

$$\Delta Д=(Д_а+Д)-Д_в=\Delta Д_д+\Delta Д_f+\Delta Д_а, \quad (5.4)$$

where separate components of this error are expressed as follows:

a) for the system with astaticism of the 1st order:

$\Delta Д_t=\frac{D}{D+K_v}Д$ - dynamic error, caused by the motion of target;

$\Delta Д_f=-\frac{K_д K_и}{D+K_v}f_p$ - error, caused by the action of interference on the radio channel;

$\Delta Д_а=-\frac{1}{D+K}\varphi_а$ - error, caused by errors in the self-contained meter;

b) for the system with astaticism of the 2nd order component are expressed by the formulas:

$$\Delta Д_д=\frac{D^2}{D^2+K_в D+K_v}Д;$$

$$\Delta Д_f=-\frac{K_в D+K_{д1}K_{д2}K_и}{D^2+K_в D+K_v}f_p;$$

$$\Delta Д_а=-\frac{D}{D^2+K_в D+K_v}\varphi_а.$$

Since the conditions of invariance are satisfied, in the general/common/total expression for the error it is not contained component caused by proper motion of flight vehicle ($Д_а$).

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S5.2. Measuring errors of range by complex range finder.

The measuring errors of range for both types of complex range finders we will seek taking into account the fact that the transmission factor of temporary/time discriminator depends on level and character of the interferences, which operate on the receiver of range-only radar. However, for each assigned interference level its average/mean value $K_{вр0}$ - constantly.

For computing the errors it is necessary to know the character of dependence $K_{вр}$ on the relation signal/noise, an to also have available the characteristics of equivalent interference $\dot{f}_{р}(t)$ at the output of temporary/time discriminator. The conclusion/output of the corresponding expressions for the convenience is carried out into the application/appendix.

Let us consider errors for both types of the range-only radar:

a) range finder with astaticism of the 1st order.

For computing the dynamic errors it is necessary to assign the law of the motion of target. Let us assume tha the component of range

is equal to:

$$Д = Д_0 + vt + \beta(t). \qquad (5.5)$$

where $Д_0$ - initial component of range D(t); v - projection of target speed on the direction flight vehicle - target; β(t) - the stationary random of function with the zero average/mean value, that reflects the random character of the displacement/movement of target.

In accordance with expressions (5.4) and (5.5) dynamic error has two components:

- *velocity error* $\Delta Д_{дv} = \frac{v}{K_0}$;

- random error with the dispersion

$$\sigma^2_{д\beta} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{j\omega}{j\omega + K_0} \right|^2 S_\beta(\omega)\, d\omega. \qquad (5.6)$$

I latter/last formula $S_\beta(\omega)$ - the spectral density of random function β(t). Let us take for an example, that spectral density $S_\beta(\omega)$ is equal to

$$S_\beta(\omega) = \frac{2\alpha_д \sigma^2_\beta}{\alpha^2_д + \omega^2}. \qquad (5.7)$$

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Then, after the computation of integral (5.6), we find

$$\sigma^2_{д\beta} = \frac{\sigma^2_\beta \alpha_д}{\alpha_д + K_0}.$$

For computing the dispersion $\sigma^2_{Д_f}$ of the error, caused by

interference $f_D(t)$, it is necessary to determine the spectral density of the latter.

On the basis (P. 25) we have, that this spectral density is expressed by the following formula:

$$S_f(\omega)=\sigma_f^2\tau_1\frac{\sin^2\frac{\omega\tau_1}{2}}{\left(\frac{\omega\tau_1}{2}\right)^2}, \qquad (5.8)$$

where τ_1 - puls duration at the output of temporary/time discriminator; σ_f^2 - dispersion, expressed by formula (P.29).

Since in accordance with equality (5.1)

$$\sigma_{Д_f}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K_п^2K_м^2}{|j\omega+K_0|^2}S_f(\omega)\,d\omega,$$

that after substitution in (5.8), we obtain:

$$\sigma_{Д_f}^2=\frac{\sigma_f^2}{K_{пр0}^2\tau_1K_0}\left(K_0\tau_1-1+e^{-\tau_1K_0}\right).$$

Taking into account that for the integrated systems ASD usually is satisfied the condition $\tau_1K_0 \ll 1$, let us expand an expression in series/row and we will be bounded by two members. Then

$$\sigma_{Д_f}^2\approx\frac{\sigma_f^2}{K_{пр0}^2}\cdot\frac{K_0\tau_1}{2}.$$

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After using the obtained expressions for signal and dispersion of interferences at the output of temporary/time discriminator [formula (P.19), (P.20) and (P.26)], when $\tau_{и}=\tau_{сан}$, it is possible to show that the dependence of the variance of error of ranging, caused by interferences on the radio channel, in the trimmed/steady-state mode/conditions takes the form:

$$\sigma^2_{Д_f}=\frac{c_1 k^2_{1п}\tau_и T K_0}{8\Delta f_{пр}}\cdot\frac{\sqrt{2}+2<q_0^2>}{[<q_0^2>]^2}-$$

for the case of square-law detection:

$$\sigma^2_{Д_f}=\frac{c_1 k^2_{1п}\tau_и T K_0}{8\Delta f_{пр}}\cdot\frac{\sqrt{2}+1,3<q_0^2>-0,51<q_0^4>}{[<q_0^2>]^2}-$$

for the case of linear detection with $q_0(t)\leqslant 1$, where q_0 - relation the signal/noise [see (P.3)], c_1 - the constant coefficient, equal to 0.93 (see the appendix), $k_{1п}$= 150 m/μs.

The dispersion of interferences at the input of detector is proportional to the passband of receiver $\Delta f_{пр}$. Then it is not difficult to show that in the assigned ratio of the square of signal amplitude at the input of receiver to the spectral density of interferences at its input N the variance of error of ranging virtually depends only on the repetition period of $\sigma^2_{Д_f}$ impulses/momenta/pulses T and on the transmission factor of servo system.

Let us determine the variance of error of ranging, caused by errors in the channel of self-contained meter φ_a. For the certainty of considerations we consider that as the self-contained meter is used the sensor of airspeed, the correlation function of its errors taking the form (see Chapter 1)

$$R_{\varphi_a}(\tau)=\sigma_w^2 e^{-\alpha_w|\tau|}, \qquad (5.9)$$

where σ_w^2 - dispersion of the fluctuations, caused by atmospheric turbulence.

Let us note that the constant component of wind in this case is not considered.

Taking into account (5.9) the spectral density of function $\varphi_a(t)$ can be defined as

$$S_{Д_a}(\omega)=\frac{2\sigma_w^2\alpha_w}{\alpha_w^2+\omega^2}.$$

Then in accordance with (5.1) variance of error $\sigma_{Д_a}^2$ is equal to

$$\sigma_{Д_a}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{Д_a}(\omega)\left|\frac{1}{j\omega+K_0}\right|^2 d\omega=\frac{\sigma_w^2}{K_0(\alpha_w+K_0)}.$$

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Taking into account (5.4) the average/mean value of the square of the error in the trimmed/steady-state mode/conditions for the system with astaticism of the 1st order takes the form

$$<[\Delta Д]^2> = [\Delta Д_{дv}]^2 + \sigma^2_{д\beta} + \sigma^2_{Д_f} + \sigma^2_{Д_ш}. \qquad (5.10)$$

In the assigned block diagram of integrated ranging system for the purpose of reaching/achievement of its maximum freedom from interference the problem consists of the selection of optimum transmission factor K_0, which is equivalent to the selection of the optimum passband of the system, which ensures best ranging to the target.

b). Measuring errors of range by integrated system ASD with astaticism of 2nd order.

Let us assume that the input effect varies as follows:

$$Д = Д_0 + vt + \frac{1}{2}at^2 + \beta(t), \qquad (5.11)$$

where a - relative acceleration of target; β - random component of range D(t).

Then the average/mean value of the measuring error of range by integrated system ASD with astaticism of the 2nd order is equal

$$\Delta Д_{дa} = \frac{a}{K_a}.$$

The methodology of the determination of the variance of error of ranging for the systems with astaticism of the 2nd order remains the same as for the systems with astaticism of the 1st order.

Formula for the variance of error, caused by the random displacements/movements of target, in the trimmed/steady-state mode/conditions taking into account (5.2) and (5.11) can be obtained in the form

$$\sigma^2_{\text{Д}\beta}=\frac{\sigma^2_{\beta}\alpha_{\text{д}}(K_{\text{п}}\alpha_{\text{д}}+K_{a})}{K_{\text{п}}(K_{\text{п}}\alpha_{\text{д}}+\alpha^2_{\text{д}}+K_{a})}.$$

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Expression for the variance of error, caused by interferences on the radio channel, in the trimmed/steady-state mode/conditions taking into account the fact that the transmission factor of correcting term is selected from the condition of the optimum character of transient process i.e. it is equal to

$$K_1=K_{\text{и}1}\sqrt{\frac{2}{K_a}},$$

it takes the form:

$$\sigma^2_{\text{Д}_f}=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\frac{\sigma^2_f\tau_1\sin^2\frac{\omega\tau_1}{2}}{\left(\frac{\omega\tau_1}{2}\right)^2}\left|\frac{K_{\text{k}}j\omega+K_{\text{и}1}K_{\text{и}2}K_{\text{и}}}{j\omega K_{\text{п}}+(K_a-\omega^2)}\right|^2 d\omega=$$

$$=\frac{\sigma^2_f}{K^2_{\text{пр}0}}\left[1+\frac{1}{2\tau_1}\sqrt{\frac{2}{K_a}}-\frac{1}{2\tau_1}\sqrt{\frac{2}{K_a}}\left\{\exp\left(\quad\tau_1\sqrt{\frac{K_a}{2}}\right)\right\}\times\right.$$

$$\left.\times\left(\cos\tau_1\sqrt{\frac{K_a}{2}}+3\sin\tau_1\sqrt{\frac{K_a}{2}}\right)\right].$$

Taking into account the fact that for the integrated systems ASD, as a rule, is satisfied condition $\tau_1\sqrt{\frac{K_a}{2}}\ll 1$ and, expanding $\exp\left(-\tau_1\sqrt{\frac{K_a}{2}}\right)$ in series/row, we will obtain

$$\sigma^2_{\text{Д}_f}=\frac{3}{2}\frac{\sigma^2_f}{K^2_{\text{пр}0}}\tau_1\sqrt{\frac{K_a}{2}}.$$

Formula for the variance of error, determined by errors in the channel of self-contained meter in the trimmed/steady-state mode/conditions taking into account (5.10), takes the form:

$$\sigma^2_{Д_в} = \frac{\sigma^2_{\omega}\alpha_{\omega}}{K_{п}(K_{п}\alpha_{\omega} + \alpha^2_{\omega} + K_a)}.$$

The general/common/total expression for the average/mean value from the square of the error of complex range-only radar with astaticism of the 2nd order in the trimmed/steady-state mode/conditions neglecting of the fluctuations o the parameters of system can be recorded as

$$<[\Delta Д(t)]^2> = [\Delta Д_{д_a}]^2 + \sigma^2_{Д_\beta} + \sigma^2_{Д_f} + \sigma^2_{Д_в}. \qquad (5.12)$$

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With $K_1 = K_{в1}\sqrt{\frac{2}{K_a}}, \quad K_п = \sqrt{2K_a}$

$$<[\Delta Д(t)]^2> = \frac{a^2}{K_a^2} + \frac{\sigma_\beta^2\alpha_д(\sqrt{2K_a}\,\alpha_д + K_a)}{\sqrt{2K_a}\,(\sqrt{2K_a}\,\alpha_д + \alpha_д^2 + K_a)} + \frac{3\sigma_f^2\tau_1\sqrt{K_a}}{2\sqrt{2}\,K^2_{вр}} + \frac{\sigma_\omega^2\alpha_\omega}{\sqrt{2K_a}\,(\sqrt{2K_a}\,\alpha_\omega + \alpha_\omega^2 + K_a)}. \qquad (5.13)$$

From expression (5.13) it is evident that the minimum of average/mean value from the square of error is provided by the optimum selection of the transmission factor of system K_a. Optimum value K_a is found from the condition of equality to zero partial derivative:

$$\frac{\partial}{\partial K_a} <[\Delta Д(t)]^2> = 0.$$

One should again note that all given above expressions for the statistical characteristics of the measuring errors of the range of comple range-only radar are obtained on the assumption that the parameters of systems are constant and formulas are valid only at the low values of disagreement/mismatch.

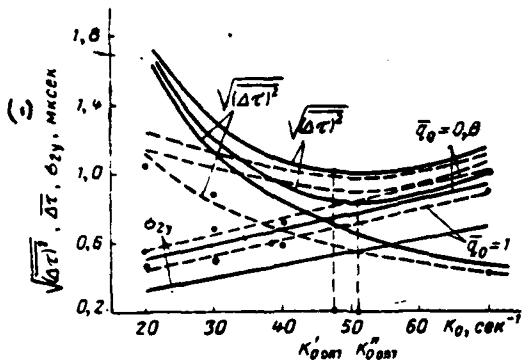

Fig. 5.3. Nonfluctuating echo signal.

Key: (1). μs. (2). s.

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For the illustration of gain in the freedom from interference of range-only radar due to the aggregation, and also for the estimation of error, caused by neglect the chance of the parameters of system Fig. 5.3-5.5 in accordance wit (5.10) and (5.12) gives the dependences of the measuring errors of range from the average transmission factor of system K_0.

In the given figures of error they have a dimensionality of time (μs), but not length (meter).

By dot-dash lines in Fig. 5.5 and dotted lines in Fig. 5.3 and 5.4 showed the curves, obtained experimentally.

Fig. 5.3 gives noise-cancelings characteristic simple system ASD with astaticism of the 1st order in the case of nonfluctuatin echo signal $(K_v(t)=K_0$ for different $\tilde{q}$, with the following values of input signals:

- relative speed of flight vehicle v=500 m/s;

- relative speed of the motion of target $v_{ц}=15$ m/s;

- speed of "constant" wind $v_{вет}=30$ m/s;

- duration of pulses $\tau_{и}=3$ μs;

- passband of receiver $\Delta f_{пр}=700$ kHz;

- pulse repetition rate $f_T=1/T=200$ Hz.

Detection in the receiver was examined by linear.

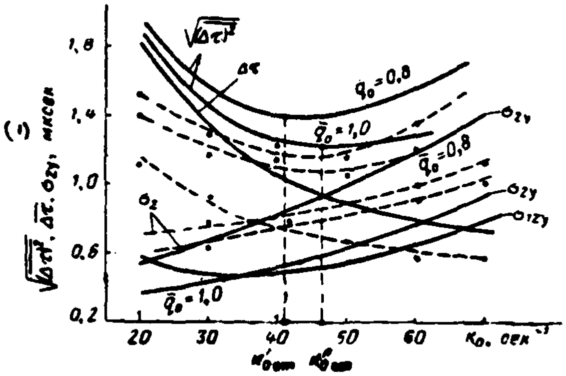

Fig. 5.4. Fluctuating echo signal.

Key: (1). μs (2). s.

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Experimental research of systems ASD was carried out by the method of the mixed simulation, with which the nonlinear part of the system (receiver and temporary/time discriminator) they were carried out in the form of the mock-ups of these devices/equipment, and the linear part of the system was the series/row of the units of electronic computer.

From the examination of the curves Fig. 5.3 it follows that the optimum values K_0 lie/rest within limits (40-50 of s^{-1}, with an increase in interferences $K_{0опт}$ it is reduced.

Fig. 5.4 gives noise-cancelings characteristic simple system ASD with astaticism of the 1st order in the case of the fluctuating echo signal (α=80 s^{-1}, m=3 σ_0), also, under otherwise equal conditions, analogous to the conditions of the previous graph. From the comparison of curves in Fig. 5.3 and 5.4 it is evident that fluctuations $K_v(t)$ call an increase both in the average/mean and fluctuation measuring errors o range to 15-50o/o. Furthermore, appears further fluctuation component of errors with dispersion $\sigma^2_{\dot{\varphi}T}$. Thus, the analysis of systems ASD without taking into account the fluctuations of the amplitudes of the echo signals gives the overstated characteristics of their freedom from interference.

Fig. 5.5 presents noise-cancelings characteristic of integrated system ASD with astaticism of the 1st order for the conditions, analogous to the conditions of the previous graphs.

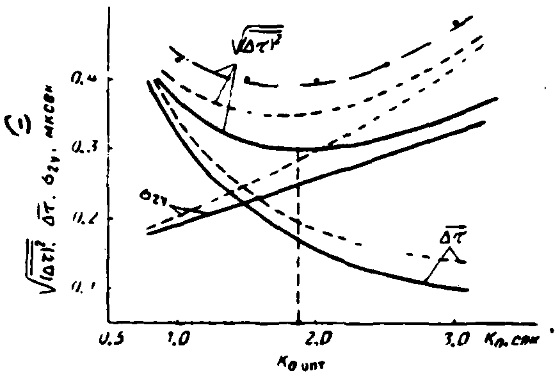

Fig 5.5. ——— nonfluctuating echo signal; - - - - - α=5 s^{-1}; -·-·-·- experiment.

Key: (1). μs. (2). s.

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The comparison of curves in Fig. 5.4 and 5.5 shows that the optimum value of the average transmission factor of integrated system ASD is substantially less than $K_{0опт}$ for the simple, and in this case for the integrated system ASD $K_{0опт}$ it is approximately/exemplarily equal to 2 s^{-1}.

Similar decrease $K_{0опт}$ causes not only an increase in the freedom fro interference of systems, but also the decrease of the

effect of the fluctuations of transmission factor on the measuring errors of range. Thus, the value of component $\sigma_{\phi\gamma}$ proves to be by an order less than others components and can be disregarded.

Under the conditions in question the variance of error, caused by interferences along the channel of self-contained meter, also proves to be lower order than value $\overline{\Delta D}$ and σ_{Δ}, and into the calculation can not enter. Limitation on further decrease of K_0 of integrated system ASD place the velocity components, caused by the displacement/movement of target and by "constant" wind.

The examination of graphs in Fig. 5.3-5.5 shows that the theoretical results will be coordinated sufficiently well with the experimental ones. An increase of the disagreements between the curves in the field of the low K_0 in Fig. 5.3 and 5.4 is explained, obviously, by the effect of the nonlinearity of the characteristic of temporary/time discriminator.

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Chapter 6.

INTEGRATED NAVIGATION AIDS.

The navigation of contemporary aircraft is provided by different devices/equipment, including self-contained and nonautonomous radio engineering systems. For the purpose of an increase in the accuracy and freedom from interference the latter frequently complex with non-radiotechnical ISD.

In this chapter will be examined some navigation aids whose action is based on the complex us of data about the navigational parameters, obtained with the help of ISD and radio engineering meters.

S6.1. Special features of the construction of complex navigational meters.

The composition of navigational complex is determined taking into account a large number of different factors. An effect on its selection have: designation/purpose, required accuracy, reliability,

freedom from interference, cost/value, etc. After the composition of complex is determined, fundamenta guides for selection of the method of the unification of meters into the system is the minimum of measuring error, sensitivity and freedom from interference, if in complex are included radio engineering meters.

Is now known a large quantity of complexes, that differs by the designation/purpose, the composition and the method of processing signals [32].

During the selection of the composition of complex, first of all, they attempt to switch on in it such meters whos statistical characteristics of errors most strong possible would differ from each other.

The accuracy of complex will be the greater, the more strongly differ the spectral characteristics of the errors of meters.

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Gain on the accuracy is reached in the simplest case due to the further filtration of the errors of primary meters. Meters themselves in this case remain constant/invariable according to the structure. The errors, caused by interferences, additionall by the ripple

filter, which ensures the mutual processing of signals (by complexing filter). The at the same time dynamic errors of radio engineering meters, if they have the low-frequency spectrum, are very little reduced due to the aggregation. Therefore in the radio engineering meters, included in the complex, it would be desirable to select passband as wide as possible. However, the expansion of the passband of the servo radio engineering system increases fluctuation error an negatively they affect the reliability of the accompaniment of signal. The appearing contradiction is solved due to the use in the complexes of the principle of the mutual correction of meters. During the aggregation not only is solved the problem of increasing in accuracy and measurement accuracy of coordinates, but also is facilitated the search for radio engineering signal, is accelerated and is simplified transition/junction to the automatic tracking. Calculation and experiment showthat most promising is the use of principle of the mutual correction of meters.

Already at the present time many systems of navigation are constructed with the use of precisely this principle.

As examples of systems with the mutual correction they can serve: the navigation aid of submarines [40], system "'SKAN" [49], inertial-Doppler system [46].

Important role during the determination of the method of the unification of meters into the complex plays the space o the necessary computations, connected with the transformation of coordinates. Errors unavoidable during calculations can adversely affect the accuracy of complex or it unjustifiably complicate. Output signal both in form of its representation and according to the utilized coordinate system must provide the maximum of conveniences to crew.

At the assigned composition of complex, the known statistical characteristics of the errors of meters, it is possible to find the structure an the parameters of complex which will make it possible to solve in the best way the problem of processing signals.

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A change in the statistical characteristics of the errors of meters in the time requires changes in the parameters of complex, and possibly, and its structure. The widespread introduction of digital onboard computers will make it possible to successfully solve this problem. Further increase in the accuracy of the solution of navigational problems should beattained not only due to an increase in the accuracy of primary meters, but also due to their rational unification into the complexes. If for the meter, not connected with

the complex, it suffices it was sufficient to be given the value of rms error, without being interested in its spectral composition, then for the meter, connected with the composition of complex, the spectral composition of errors is determining during its selection.

Thus, if in the errors of non-radiotechnical meter is included slowly varying component even sufficient high value, it in effect completely will be reduced in the output signal in the process of the mutual processing of signals in the complex.

Below as an example will be examined two complexes, that realize the principle of mutual correction.

§6.2. Two-coordinate integrated system of short-range navigation.

The integrated systems of the short-range navigation of aircraft prove to be, as a rule very simple. As the example, which makes it possible to explain the fundamental principles of the unification of meters into the complex, let us consider integrated two-coordinate system "SKAN" [49]. retaining the composition of the meters connected with it, we will consecutively/serially compose the block diagram of system in order as the final result in the complex to come to light/detect/expose, as is realized the principle of the mutual correction of meters during the rational construction of system.

With the composition of complex they are connected:

- the azimuth and range finding nonautonomous radio-navigation system, which makes it possible to measure the azimuth of ground-based radio beacon γ_p and slant range to it $Д_p$;

- sensor of airspeed;

- course system.

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The vector of airspeed v of aircraft (Fig. 6.1) can be decomposed by two methods:

along the axes, connected with the lines of position of radio system;

- along the axes, connected with the direction of track.

Fig. 6.1 depicts the rectangular coordinate system $x_з o_з y_з$, moreover $o_з y_з$ is directed along the magnetic meridian. Track is

characterized by predetermined course $\psi_з$.

Let us select coordinate system $\eta_a O_a \xi_a$, beginning of which coincides with the starting point of route O_a, and axis $O_a\xi_a$ coincides with the assigned track, i.e., it passes through the target Ts.

Let us decompose thevector of the airspeed of aircraft v on the components along the axes of the aircraft system of coordinates

$$v_\eta = v \sin \Delta\psi_a;\ v_\xi = v \cos \Delta\psi_a, \qquad (6.1)$$

where

$$\Delta\psi_a = \psi_з - \psi_a; \qquad (6.2)$$

$\psi_з$ - the predetermined course; ψ_a - measured value.

In this case it is assumed that the longitudinal axis of aircraft coincides with the direction of speed.

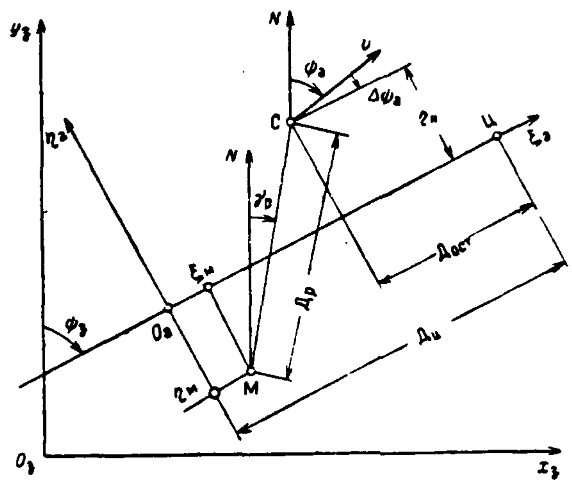

Fig. 6.1.

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For the comparison of signals ISD (course system and the sensor of airspeed) them it is necessary these signals to convert into the signals of position, i.e. to integrate.

In the device/equipment of comparison from ISD will come the signals:

$$\left.\begin{aligned} \xi_{a\text{н}} &= \int_0^t v(\tau)\cos\Delta\psi_a(\tau)\,d\tau, \\ \eta_{a\text{н}} &= \int_0^t v(\tau)\sin\Delta\psi_a(\tau)\,d\tau. \end{aligned}\right\} \qquad (6.3)$$

Radio engineering system when $\eta_м$ and $\xi_м$ (coordinate of beacon in system $\eta_a o_a \xi_a$) substantially more than flight altitude of aircraft H_c, makes it possible to obtain the same coordinates:

$$\left.\begin{aligned}\xi_p &= \xi_м + Д_p \cos(\psi_в - \gamma_p),\\ \eta_p &= \eta_м + Д_p \sin(\psi_в - \gamma_p),\end{aligned}\right\} \qquad (6.4)$$

where $Д_p = Д + \Delta Д$, $\gamma_p = \gamma + \Delta\gamma$,

$\Delta Д$, $\Delta\gamma$ - the error of radio engineering system.

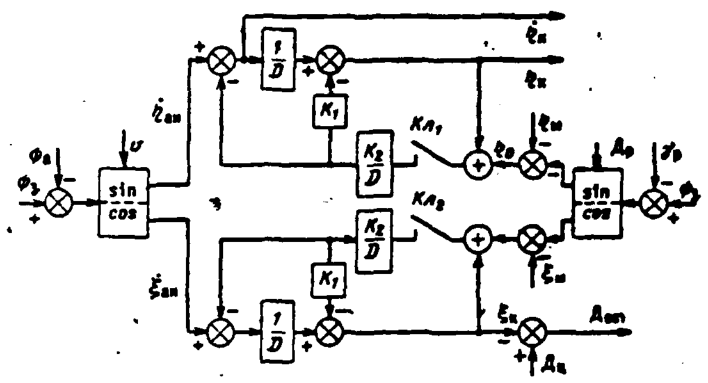

Fig. 6.2.

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Solution of equations (6.3) and (6.4) (when $\xi_{ак}=\xi_{ар}$ and $\eta_{ак}=\eta_{ар}$) gives the possibility to obtain the signals, suitable for the comparison in the filter general/common/total for them, i.e., to construct integrated system without the mutual correction.

The block diagram of this complex is given in Fig. 6.2. System can work in the mode/conditions of periodic or incidental correction, since filter possesses the memory on the constant measuring error of speed.

In each of the channels are used identical filters. Let us consider one of the filters, for example working in the channel of the

measurement of the lateral divergence of aircraft η_{κ} and of derivative this divergence $\dot{\eta}_{\kappa}$. If we to the input of filter feed the signals

$$\dot{\eta}_{a\kappa} = \dot{\eta}_a + \Pi_a, \quad \eta_{\text{p}} = \eta_a + \Pi_{\text{p}},$$

where η_a and $\dot{\eta}_a$ - true value of coordinate and its derivative; Π_a and Π_{p} - errors of the signals of self-contained and radio engineering systems, determined at the input of filter, then signals at its output will be described by the equations

$$\eta_{\kappa} = \eta_a + \frac{D\Pi_a}{N_{\Phi}(D)} + \frac{K_2(DK_1 + 1)\Pi_{\text{p}}}{N_{\Phi}(D)}, \qquad (6.5)$$

$$\dot{\eta}_{\kappa} = \dot{\eta}_a + \frac{D(D + K_1K_2)\Pi_a}{N_{\Phi}(D)} + \frac{K_2D\Pi_{\text{p}}}{N_{\Phi}(D)}. \qquad (6.6)$$

In equations (6.5) and (6.6)

$$N_{\Phi}(D) = D^2 + K_1K_2D + K_2.$$

As can be seen from thegiven equations, in the trimmed/steady-state mode/conditions neither to the measured coordinate nor on its derivative the constant errors of self-contained sensors an effect have. Consequently, the constant components of signals at the outputs of the integrators of correction (integrators with the transmission factor K_2) are equal to these errors and it is possible to realize a periodic or incidental correction of the computers of path (integrators with the transmission factor, equal to one in Fig. 6.2). The interesting special feature/peculiarity of filter is the fact that the integrator of dead reckoning is connected with the filter.

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Deficiencies/lacks in the complex should be considered:

- the absence of the correction of the radio engineering servo systems;

- unavoidable losses in the accuracy of the determination of the coordinates of aircraft due to the errors of the computer (SRP), which decides equations (6.4).

Fo the realization of the principle of the correction of radio engineering meters D and γ by the signals of self-contained meters it is possible to decompose the vector of airspeed v on the components: coinciding with the direction to the radio beacon and to it perpendicular.

From Fig. 6.3 it is not difficult to find

$$\left.\begin{aligned} v_R &= \frac{d Д(t)}{dt} = v\cos(\varphi_a - \gamma_r), \\ v_\gamma &= Д\,\frac{d\gamma(t)}{dt} = v\sin(\varphi_a - \gamma_p) \end{aligned}\right\} \qquad (6.7)$$

or

$$\frac{d\gamma(t)}{dt} = \frac{v}{Д}\sin(\varphi_a - \gamma_p). \qquad (6.8)$$

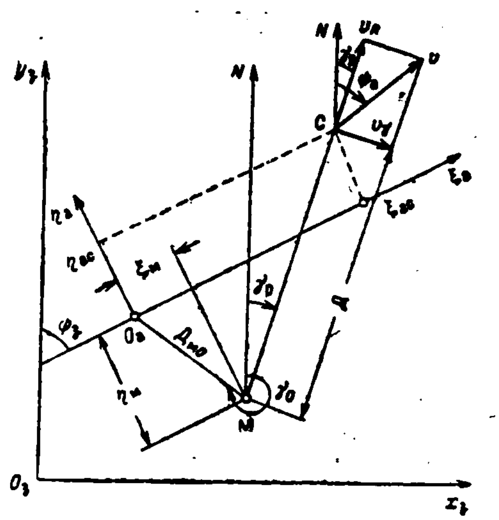

Fig. 6.3.

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Signals (6.7) and (6.8) are suitable for the correction of radio engineering meters on the speed, but the difficult of their obtaining lies in the fact that $\gamma_м$ and D is previously unknown and it is possible to obtain them only after the entrance of radio engineering signals.

Therefore it is expedient to convert the rectangular coordinates of aircraft ξ_a and η_a counted by self-contained system (see Fig. 6.3) into the polar system with the beginning at the point of setting beacon M and to carry out a correction of meters γ and D with respect to position.

position.

If the coordinate of beacon in system $\eta_a O_a \xi_a - \xi_M$ and η_M, then as a result of dead reckoning in two mutually perpendicular directions it is possible to define the coordinates of beacon on the aircraft as

$$\left.\begin{aligned} \xi_{MC} &= \xi_a - \xi_M = \int_0^t v(\tau)\cos\Delta\psi_a(\tau)\,d\tau - \xi_M, \\ \eta_{MC} &= \eta_a - \eta_M = \int_0^t v(\tau)\sin\Delta\psi_a(\tau)\,d\tau - \eta_M. \end{aligned}\right\} \quad (6.9)$$

Comparing (6.9) with (6.4) we see that they coincide with the only the difference, that (6.9) are determined the coordinates of beacon relative to aircraft, and (6.4) - rectangular coordinates of the aircraft through coordinates $Д_p$ and γ_p, measured by radio engineering system. Taking into account of the aforesaid, equation of relation for the coordinates of beacon it is easy to obtain in the form

$$\left.\begin{aligned} \xi_{MC} &= Д_a\cos(\psi_a - \gamma_{aM}), \\ \eta_{MC} &= Д_a\sin(\psi_a - \gamma_{aM}). \end{aligned}\right\} \quad (6.10)$$

Solving together equations (6.9) and (6.10), it is possible to obtain signals $Д_a$ and γ_{aM}, suitable for the correction of the servo systems of the aircraft meters of the azimuth and range finding system on the position.

The coordinates, measured by the corrected azimuth and range finding system, can be, on one hand, used for the indication of the

position of aircraft, and on the other hand - for the correction of the computers of path. It is obvious that inverse transformation of coordinates $Д_p$ and γ_p into coordinates ξ_p and η_p must be realized in accordance with equalities (6.4).

Using signals η_p and ξ_p, it is possible to form signals for the self-contained computers of path $\Delta v_{\eta a}$ and $\Delta v_{\xi a}$, thereby to raise the accuracy of the solution of navigational problem in the absence of radio signals.

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As the signals, convenient for aircraft handling, it is expedient to use signals $\eta_{ак}$, $\xi_{ак}$ and $\eta_{ак}$.

Thus, the functional diagram of the integrated system of the short-range navigation, in which is realized the principle of mutual correction, can be represented in the form, depicted in Fig. 6.4. Here SPK - the diagram of the translation of coordinates, BFSK - the shaping unit of correction, I - integrators, VU - subtractor.

Diagram does not need further explanations. Let us simply note that versions examined above of the unification of meters by no means exhaust all having the capability.

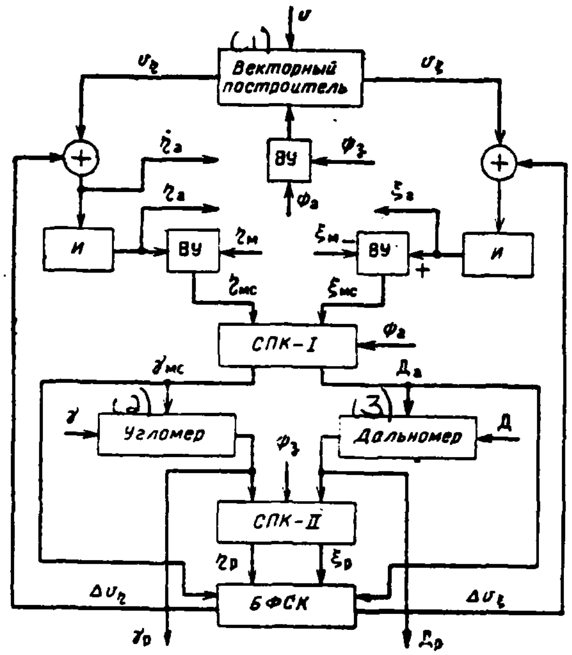

Fig. 6.4.

Key: (1). Vector builder. (2). Goniometer. (3). Range finder.

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Thus remained the not included by circuit correction course system, although the error of system exerts a substantial influence on the errors of complex, but, in the principle, it is possible to correct gyroscope.

The important advantage of the system in question is the fact that the signals of the correction of radio engineering meters $\gamma_{ам}$ and $Д_а$ continuously enter the input of the meters of range D and angle γ. Therefore there is no need for realizing a repeated search for signals (after the loss of accompaniment) on the entire distance of the measurements D and γ. Is sufficient to realize a search within the limits of the maximum errors of the transmission/delivery of preliminary data $\gamma_{ам}$ and $Д_а$ respectively.

S6.3.

Complex Doppler and is inertial - Doppler systems.

Very wide application in the aviation find the Doppler meters of the vector of ground speed and drift angle (DISS).

At present DISS are used both independently and in complex and by inertial systems. In the latter case DISS it is used for purposes of the correction of inertial systems on speed [16]. An increase in the velocities, accelerations and flight altitudes of aircraft continuously raises requirements for the parameters of DISS. Thus, for instance, increase of flight altitude two times, with the retention/preservation/maintaining of previous sensitivity of the meter of frequency, requires an increase of the power of transmitter

four times with the continuous and eight times during the pulse radiation/emission (in DISS with the external coherence). Therefore every possible increase in the sensitivity of measuring systems DISS remains urgent problem. The effective method of the solution of the problem indicated is the use of signals of velocity or acceleration of aircraft, obtained with the help of other self-contained meters.

The most fully indicated problem is solved in inertial-Doppler systems with the mutual correction. The unification of inertial system and DISS from the point of view of effect, attained by integration, is most appropriate. The spectrum of errors DISS is sufficiently wide, and the errors of inertial system are very narrow-band, but they are accumulated in the course of time.

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The fact indicated was used in the inertial systems with the correction on velocity [16]. On the work of inertial system the fluctuation errors of the Doppler signal of a practical effect do not have, but the average/mean value of signal, which corresponds with an accuracy to dynamic error to the true value of velocity, raises the accuracy of inertial system. A deficiency/lack in the complex is the absence of the feedback loops of inertial system with the meters (for example, by the servo frequency meters) of DISS. Although during the

use of DISS as the means the corrections of the inertial system of requirement for the minimally necessary signal-to-noise ratio can be lowered/reduced, this, as it will be shown below, it is not optimum resolution of problem.

In [46] is described the system, which uses principle of mutual correction. Inertial system is corrected the signals velocity, by measured by DISS. At the same time the signal of ground speed, taken from the output of the first integrator of inertial system, controls the heterodyne of the servo frequency meter of Doppler station, and antenna DISS occupies position in accordance with the value of drift angle, which is determined by inertial system. Thus in the complex the principle of mutual correction is realized both along the channel of the measurement of the modulus/module of the vector of ground speed and along the channel of drift angle. During this unification of systems most fully is used the possibility for increasing the accuracy of complex and sensitivity of meters DISS.

The problem of an improvement in the sensitivity of DISS can be solved, although and incompletely, with the use of the very simple and always available on board the aircraft sensors, in particular the sensor of airspeed (DVS).

A) complex frequency meter with the use of signals of the sensor of

airspeed (DVS).

Signal at the output DVS is conveniently represented in the form (see Chapter 1)

$$v = w - \Delta w, \tag{6.11}$$

where w - ground speed of aircraft; Δw - error in the measurement of ground speed, caused, first of all, by the "constant" wind u.

As can be seen from (6.11), signal DVS carries information about the ground speed, although with the large error.

The block diagram of the meter of Doppler frequency is given in Fig. 6.5. This diagram differs from widely known frequency meter [31] in terms of introduction to the control circuit of the adder, to one input of which is fed the voltage/stress from the output of the integrator of the diagram of tracking the Doppler frequency, and on the second - voltage/stress, proportional to airspeed. The block diagram of frequency meter is given in Fig. 6.6a.

Here $F_{д0}$ - average/mean value of Doppler frequency, proportional to the modulus/module of the vector of ground speed;

$\Delta f_п$ - disturbances/perturbations, which operate on the input of system are caused by random character of useful signal and by

receiver noise;

$\frac{K_{чд}}{T_{д}D+1}$ - transfer function of the FM discriminator;

$K_{чд}$ - transmission factor of the FM discriminator;

$K_{и}$ - transmission factor of integrator;

$K_{у}$ - transmission factor of the controlled heterodyne.

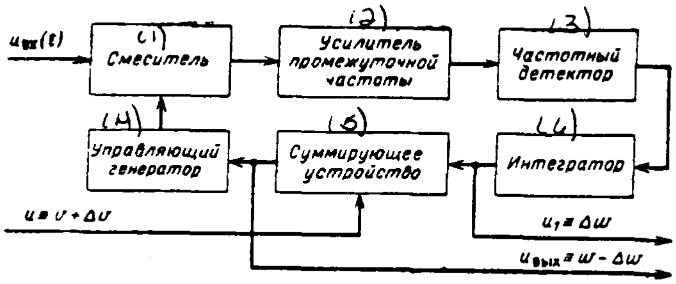

Fig. 6.5.

Key: (1). Mixer. (2). Intermediate-frequency amplifier. (3). FM discriminator. (4). Controlling generator. (5). Adder. (6). Integrator.

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It is interesting to note that output potential of integrator (at the input of adder) is proportional to the longitudinal component of wind u_t. Below we will show this.

For simplification in the analysis the structural of the diagram of frequency meter is conveniently converted. Let us take into account that $F_{до} = w$; $u_{вых} = w - \Delta w$, where Δw - error of the servo filter (m/s), and let us disregard/neglect the inertness of the controlled heterodyne.

With any physical analog of output signal the structural of diagram we convert to the form, shown in Fig. 6.6b, assuming/setting $K_y=1$.

Here: δ_1 - disturbances/perturbations, caused by radio interferences and random character of input signal;

$w_{вых}$, $u_{вых}$ - measured values w and u_t respectively.

The obtained diagram indicates that the system in question relates to the class of systems with the position correction.

Output signal of the servo system

$$w_{вых}=w+\frac{W_\Phi(D)}{1+W_\Phi(D)}\delta_1-\frac{1}{1+W_\Phi(D)}\Delta w. \quad (6.12)$$

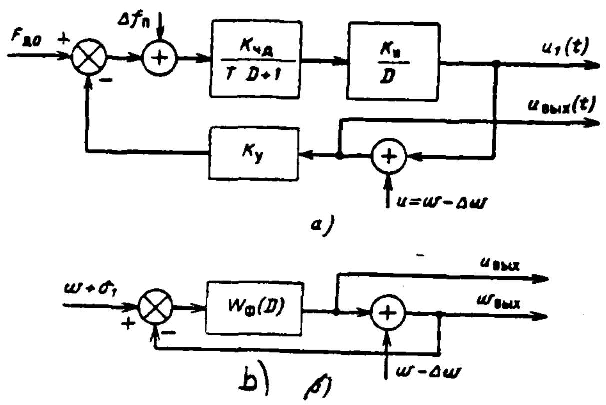

Fig. 6.6.

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Moreover, that

$$W_{\Phi}(D)=\frac{K_v}{D(T_д D+1)},$$

where K_v - transmission factor of the duct/contour of frequency meter on the velocity; T_{Φ} - time constant of the filter of frequency discriminator.

$$\Delta\omega_{вых}=\frac{K_v}{A(D)}\delta_1-\frac{D(T_д D+1)}{A(D)}\Delta\omega, \qquad (6.13)$$

where

$$A(D)=T_д D^2+D+K_v.$$

Then by shape, in complex frequency meter should be considered only the components of errors, caused:

- by the structure of Doppler signal and by radio interferences;

- by its own errors VS;

- by change in time and space of laminar flows of air (by changes in the "constant" wind);

- by disturbances/perturbations, which operate at the input DVS, first of all due to turbulence of the atmosphere.

Let us find expressions for the dispersions of the errors indicated.

Let that or in another manner find the spectral density of the fluctuations of signal at the output of the FM discriminator (to the ripple filter).

After this it it is easy to convert to the input of system and to lead to scale and dimensionality of the spectral density of the measured value, i.e., m²/s.

If we designate through $S_f(\omega)$ spectral density of output disturbances of the FM discriminator, then the spectral density of the disturbance/perturbation, led to the input (δ_1 in Fig. 6.6b),

$$S_\delta(\omega)=\frac{S_f(\omega)}{K_\text{д}^2}K^2,$$

where K - proportionality factor, which connects Doppler frequency

with the velocity

$$K=\frac{c}{l_0}=\frac{w}{F_{д0}}.$$

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Spectral density $S_\delta(\omega)$ can be identified with the white noise, therefore, as it is easy to be convinced, the variance of error, caused by noises and random character of signal,

$$\sigma_\delta^2=\frac{S_\delta(0)}{2}K_v. \qquad (6.14)$$

The component of error, caused by its own errors DVS, is the slowly varying function of time. It composes 1-2o/o of v and in the trimmed/steady-state mode/conditions can be disregarded. Expression for the variance of error of the measurement of the modulus/module of the vector of ground speed by the complex servo frequency meter, caused by atmospheric turbulence, after using formulas (1.58), (6.13), can be recorded in the form

$$\sigma_T^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\sigma_u\alpha_u(\omega+T_д^2\omega^4)\,d\omega}{|(-T_д\omega^2+j\omega+K_v)(\alpha_u+j\omega)|^2},$$

where $\alpha_u=\frac{v}{L},\cdot L$ - three-dimensional/space scale of turbulence.

After leading integration, we will obtain

$$\sigma_T^2=\sigma_u^2\frac{[T_д(\alpha_u+K_v)+1]\,\alpha_u}{T_д\alpha_u^2+K_v+\alpha_u}. \qquad (6.15)$$

We will be bounded to the qualitative analysis of the obtained relationship/ratio. the error of complex frequency meter the less, the higher the velocity of aircraft. For the complex servo frequency

meters usually is satisfied condition $\alpha_u/K_v \gg 1$ (α_u it is of the order of ones; and $K_v < 0{,}1$ s^{-1}). Therefore

$$\sigma_T^2 \approx \sigma_u^2.$$

As far as the numerical values of the error, caused by atmospheric turbulence, are concerned, for the absolute majority of the cases it proves to be negligibly low. Thus, $\sigma_u = 3$ m/s corresponds to flight in the strongly disturbed atmosphere. Usually $\sigma_u < 1$ m/s.

Laminar flows of air ("constant" wind) have an effect on the work of complex frequency meter only if its longitudinal component varies its value. Of this it is easy to be convinced.

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The component of the error, caused by wind, is described by the equation

$$\Delta w_u = \frac{Du_t}{T_1 D^2 + D + K_v}. \tag{6.16}$$

With $t \to \infty$ and $u_t = \text{const}$ $\Delta w_u \to 0$. The three-dimensional/space and temporary/time gradient of wind also is very low, and the error of complex frequency meter in the trimmed/steady-state mode/conditions

$$\Delta w_{u\,\text{уст}} = \frac{\dot{u}_t}{K_v}$$

even with $K_v < 0{,}1$ s^{-1} is low. Vector in the value and the direction in flight of aircraft will vary very by the slowly and servo frequency meter even with its very large inertness will be tracked virtually

without the error. However, to make the conclusion that the wind effect must not be considered during the identification of the parameters of the servo frequency meter prematurely. With a change in the direction of flight varies the longitudinal component of wind even with $|w|$=const.

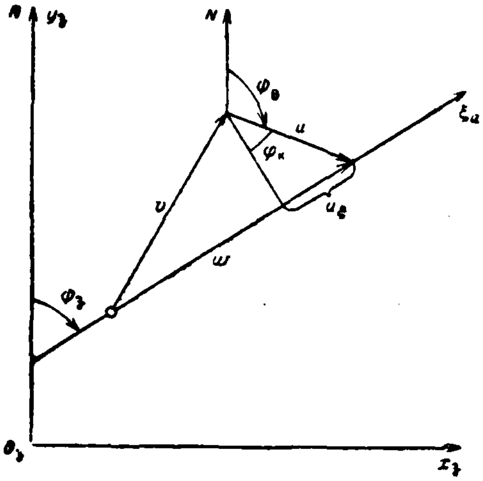

Fig. 6.7.

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Let us find dependence u_ξ on a change in the direction of flight. For this let us turn to Fig. 6.7.

The direction of flight in fixed coordinate system $x_з$, $o_з$, $y_з$, y of which axis $o_з y_з$ coincides with the direction of magnetic meridian, it is assigned by magnetic heading $\Psi_з$. Wind direction is characterized by wind angle $\Psi_в$.

Using auxiliary angle $\varphi_к$, not difficult for the longitudinal component of wind u_ξ to obtain the dependence

$$u_\xi = u \cos(\Psi_в - \Psi_з).$$

Moreover, that Ψ_B and u do not vary, but aircraft alters course, i.e., $\Psi_3=\Psi_3(t)$. Then time derivative (gradient of longitudinal component)

$$\dot{u}_{\xi}=\frac{du_{\xi}}{dt}=u\sin[\Psi_B-\Psi_3(t)]\dot{\Psi}_3(t). \qquad (6.17)$$

It is known that maximum permissible rate of change in course $\dot{\Psi}_{макс}$ is connected with permissible overload n with the formula

$$\dot{\Psi}_{макс}=\frac{ng}{w}\frac{рад}{сек},$$

where g=9.81 m/s² - acceleration of the earth's gravity.

Under unfavorable conditions when

$$\sin(\Psi_B-\Psi_3)=1$$

the error of complex frequency meter will be determined by formula [1]

$$\Delta w_{\Psi}=\frac{ung}{wK_v}=\frac{a_u}{K_v}. \qquad (6.18)$$

where a_u - the "acceleration" of wind, caused by a change in the direction of flight with the maximum transverse acceleration n.

FOOTNOTE [1]. It is assumed that the parameters DISS at the bank do not vary. ENDFOOTNOTE.

Quantitative evaluation/estimate Δw_{Ψ} is hindered/hampered, since u, Ψ_B, Ψ_3 must be considered as random values. Let us take: u=10 m/s; n=1.5w=300 m/s. Then

$$\Delta w_{\Psi}=\frac{0,5}{K_v}.$$

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For simple frequency meter are characteristic two components of the error: fluctuation and dynamic. The dispersion of first component is determined also by formula (6.14), and the second in the trimmed/steady-state mode/conditions will be determined by the formula

$$\Delta w_a = \frac{a_c}{K'_a},$$

where a_c - the longitudinal acceleration of aircraft.

Complex frequency meter can provide gain on the sensitivity, if

$$a_u < a_c.$$

For the aircraft, which have high velocities, are characteristic large longitudinal accelerations. At the same time to the transverse overloadings there are harsh norms for different aircraft types. Therefore, comparing formulas (6.18) and (6.15), it is possible to draw the conclusion that, apparently, the aggregation of DISS with the sensor of airspeed will prove to be the more advantageous it will give the greater the gain in the freedom from interference, than for the more high-speed aircraft it is intended. Together with the fact that the band DISS can be throttled/tapered approximately/exemplarily by an order, there are two further advantages.

The first lies in the fact that the search for signal in the

frequency in the integrated system can be produced only in the range of frequencies, determined by the range of a change in the modulus/module of the vector of wind but not in the range of the possible velocities of aircraft. This also leads to an improvement in the characteristics of DISS. The second consists in the fact that the complex frequency meter simultaneously with the measurement of the modulus/module of vector of path velocity can measure the longitudinal component of the vector of wind.

On the basis of structural diagram in Fig. 6.6b, after taking into account (6.12), we will obtain

$$u_{вых}=\frac{K_v}{T_2D^2+D+K_v}(u_\xi+\delta_T+\delta_1), \qquad (6.19)$$

where δ_T - error of DVS, caused by turbulence. Thus, in the complex frequency meter there is no need for for the calculator of longitudinal wind. The well smoothed signal is obtained automatically.

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With the correction of the servo frequency meter by the signals of the sensor of the airspeed coefficient K_v can be reduced 5-10 times. An even larger gain can be obtained during the use for the same targets of the signals of inertial system.

B) The inertial-Doppler systems.

There are at least two methods of aggregation DISS and inertial navigation systems. Using the first method, most widely illuminated in the literature (for example, see [16]), inertial system is corrected by the signals of Doppler speed/rate meter. Using second method [46] together with the correction of inertial system on the velocity, into the servo frequency meters are introduced the signals of acceleration or the velocities, obtained with the help of the correctable inertial system. According to above classification accepted similar complexes must be related to the so-called mutually correcting systems. In whatever coordinate system worked inertial system, the signals of velocity (acceleration) obtained with its aid can be converted to the system of coordinates of DISS and introduced into the dissent of the frequency meter.

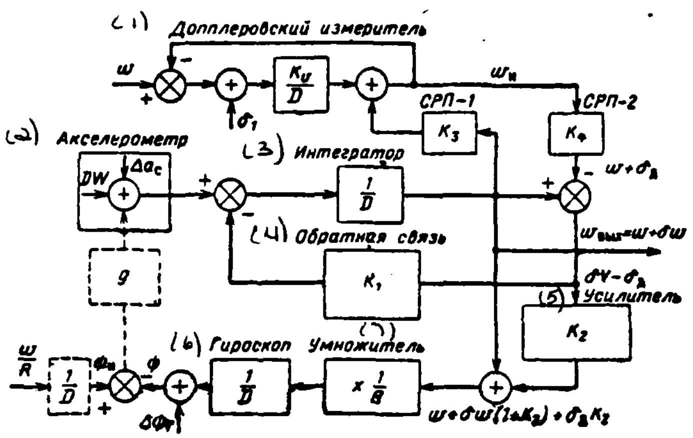

Fig. 6.8.

Key: (1). Doppler meter. (2). Accelerometer. (3). Integrator. (4). Feedback. (5). Amplifier. (6). Gyroscope. (7). Multiplier.

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For the correction of inertial system it is necessary to carry out reverse/inverse translation of the signals of the velocity, obtained with the help of the complex frequency meter, into the coordinate system in which works inertial system.

Simplest complex proves to be during the use of great-circle type inertial systems. Here translation is of the agreement of the scales of signals by determining the coupling coefficients.

For the qualitative analysis of the dynamic properties of the inertial-Doppler systems of the mentioned two types let us consider the generalized, simplified circuit of one channel. The servo frequency meter can be corrected by the signals of velocity or acceleration. From the point of view of the dynamic properties of complex both methods are completely equivalent, since the signal of the velocity in the inertial system is obtained by integrating the signals of acceleration, obtained with the help of the accelerometer. We shall examine version with the correction of frequency meter by the signal of velocity, as this is assumed in [46].

The block diagram of the mutually correcting system is given in Fig. 6.8. Examination we will conduct only for one channel.

Accelerometer measures the value of horizontal acceleration $a_x = Dw$, where w - true airspeed. Its own error of accelerometer is characterized by signal Δa_c. Platform always has the specific angle with the horizon/level γ; therefore accelerometer measures the component of the acceleration of gravity g sin $\gamma \sim\sim g\gamma$. Further from the signal of accelerometer must be subtracted calculated by special SRP components of centripetal and Coriolis acceleration. For simplification in the analysis these circuits are not shown. The

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errors of SRP in the first approximation, can be taken into account in signal Δa_c. Further the signal of accelerometer enters the integrator included by feedback. Because of this connection/communication are damped the natural oscillations of the systems, which have the period of Schuler $T_{\text{ш}}$=84.4 min. For the decrease of the error in the system, caused by the accelerations of flight vehicle, into it are introduced the signals, measured by DISS. Before the input/introduction these signals with the help of the computer SRP-2 are counted over and are led to the required scale (in the case in question one should assume K_4=1).

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At the input of servo system is applied the exciting signal, caused by random character of the echo signal and by receiver noise δ_1.

The signal of Doppler meter $w+\delta_{\text{и}}$ is compared with the signal of velocity, manufactured by the inertial system $w+\delta w$. The forming as a result of comparison (subtraction) error signal $\delta w-\delta_{\text{и}}$, on one hand, damps system (because of the presence of component δw), and on the other hand, through the amplifier with the factor of amplification K_2 corrects the gyroscope whose position is assigned also by ~~IS by~~ the signal of velocity w.

The need for change of the position of the axis of gyroscope with signal w is connected with a change in the position of local vertical line in the inertial coordinate system with the displacement/movement of aircraft. The angular velocity of gyroscope for the retention of the true vertical must be equal to

$$\dot{\Psi}_{r}(t)=\frac{w(t)}{R},$$

where R - radius of the Earth.

With a change in the coordinates of aircraft varies the latitude (longitude/length) of the place

$$\psi_{u}(t)=\frac{\int w(t)\,dt}{R}. \qquad (6.20)$$

Therefore the error in the determination of vertical line will be equal to

$$\gamma=\psi_{u}-\psi.$$

Gyroscope has its own drift, characterized by signal $\Delta\psi_{r}$. The correction of the servo frequency meter DISS is provided through the SRP 1.

For simplification in the analysis it is accepted that SRP is inertia-free and has a transmission factor $K_{3}=1$.

Equation for the measuring error of the velocity can be obtained in the form

$$\delta w=\frac{D\Delta a_c+gD\Delta\psi_r+\left(K_1D+\frac{K_2 g}{R}\right)\delta_{Д}}{D^2+DK_1+(1+K_2)\frac{g}{R}}. \qquad (6.21)$$

The further equation which should be considered during the analysis of an inertial-Doppler system, is equation for the output signal of DISS, which contains error $\delta_{Д}$.

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It easily is obtained on the basis Fig. 6.8:

$$\delta_{Д}=\frac{D(1-K_3)}{D+K_4}w+\frac{K_4}{D+K_4}\delta_1+\frac{K_3D}{D+K_4}\delta w. \qquad (6.22)$$

The error analysis of corrected and mutually-corrected systems can be led, assuming/setting in equation (6.29) K_3=0 or K_3=1. To system with the simple correction corresponds K_3=0, to system with the mutual correction K_3=1.

Assuming/setting K_3=0, equation for the error of corrective command $\delta_{зн}$ we will obtain in the form

$$\delta_{дн}=\frac{Dw}{D+K_4}+\frac{K_4\delta_1}{D+K_4}. \qquad (6.23)$$

Disturbance/perturbation δ_1 can be identified with the white noise, and the input effect w(t) can be represented in the form of the determined function of the time:

$$w=w_0+a_ct. \qquad (6.24)$$

Conservative value of velocity error of Doppler system produces the appearance of the dynamic error of the complex

$$\delta_{\text{до}} = \frac{a_c K_2}{(1 + K_2) K_v}. \qquad (6.25)$$

Since K_2>>1, then

$$\delta_{\text{до}} \approx \frac{a_c}{K_v}. \qquad (6.26)$$

Thus, dynamic error DISS virtually without the changes passes to the output of integrated system.

Equation for the error of the corrected meter of Doppler system (with K_2=1)

$$\delta_{\text{дк}} = \frac{D}{D + K_v} \delta w + \frac{K_v}{D + K_v} \delta_1. \qquad (6.27)$$

Equation (6.27) shows that the system with the mutual correction proves to be invariant to the velocity and its errors depend only on the errors of elements/cells Δa_c, $\Delta\psi_r$ and external disturbance/perturbation δ_1. It is natural that the introduction of the signal of correction to the servo frequency meter varies the dynamic properties of complex.

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After substituting in (6.21) and (6.22) equation (6.27) and after solving them together, we will obtain for the mutually-corrected system of equations the measuring error of the velocity:

$$\delta w_{\text{к}} = \frac{(D+K_v)D}{N_{\text{к}}(D)}\Delta a_c + \frac{g(D+K_v)D}{N_{\text{к}}(D)}\Delta\psi_r + $$
$$+ \frac{RK_vK_1D + K_vK_2g}{N_{\text{к}}(D)}\delta_1, \qquad (6.28)$$

where

$$N_{\text{к}}(D) = D^3 + (K_v + K_1)D^2 +$$
$$+\left(K_1K_v + \frac{g}{R} + \frac{K_2g}{R}\right)D + (1+K_2)\frac{K_vg}{R} -$$

- characteristic equation of system.

For the comparison of two types of systems let us represent disturbances/perturbations in the form (1.46):

$$\left.\begin{aligned} \Delta a_c &= \Delta a_0 + a_v t, \\ \Delta\psi_r &= \Delta\psi_0 + b_1 t. \end{aligned}\right\} \qquad (6.29)$$

In the system with the simple correction conservative values of the components of the measuring error of the velocity

$$\left.\begin{aligned} \delta w_{\text{на}} \approx \frac{a_c}{K_v}, \quad \delta w_{\text{н.w}} = \frac{Ra_v}{(1+K_2)g}, \\ \delta w_{\text{н}\psi} = \frac{Rb_1}{(1+K_2)g}. \end{aligned}\right\} \qquad (6.30)$$

In the system with the mutual correction components $\delta w_{\text{нw}}$ and $\delta w_{\text{н}\psi}$ remain without the changes, but is absent dynamic error, i.e., $\delta w_{\text{на}} = 0$. The fact indicated, is at first glance, it can seem by unessential, since by increase K_v it is possible to make $\delta w_{\text{на}}$ value as low as conveniently possible. However, with an increase of the level of interferences at the output of frequency meter grow the fluctuation and dynamic errors DISS [1].

FOOTNOTE [1]. Dynamic error grows due to the decrease of the transmission factor of FM discriminator $K_{\text{чд}}$ and, consequently K_v. ENDFOOTNOTE.

On the specific interference level unavoidably occurs the loss of the accompanied signal. In the mutually correcting system the dynamic error is absent and limitations to decrease K_v virtually can be limited only by the time of setting process.

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Reducing K_v, it is possible to attain what conveniently decrease of the fluctuation error in the feedback loop of frequency meter and thereby to raise the reliability of the accompaniment of Doppler signal. The fact that decrease K_v improves the filtration of interferences by integrated system, apparently, does not have important value, since inertial system itself is the narrow-band filter, which well smooths out radio interferences.

Further development of systems with the mutual correction can be systems with the frequency meters, which do not have their own integrators. Here the role of comparator, which realizes subtraction of signals δw and $\delta_{\text{д}}$, will perform the FM discriminator, and control of the frequency of heteroine will be realized by signals of the

integrator of inertial system.

The principle of mutual correction is used also for the servo system, which measures the drift angle.

Inertial system and DISS determine this angle. Consequently, it is possible to use both meters in the system of mutual correction.

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Appendix.

Analysis of the transmission properties of temporary/time discriminator.

Temporary/time discriminator during the analysis of integrated systems was dynamic inertia-free component/link with random coefficient $K_{\text{вр}}(t)$, distributed according to the normal law, and by additive noise $f_{\text{р}}(t)$.

Let us give the substantiation of this representation and let us find the appropriate statistical characteristics of the values indicated.

Voltage on the input of the radio receiver of system ASD is the mixture of useful signal and interferences. Interferences can be caused, for example, by the inherent noise of receiver, by the fluctuations of the echo signals on the amplitude, etc. Similar

interferences in their majority are normal and have continuous spectrum up to the very high frequencies. The width of the spectrum considerably exceeds the bandwidth of input circuits of receiver. Subsequently the set of the named noises (besides interferences due to the fluctuations of the amplitudes of the echo signals) we will characterize by the only parameter - spectral density N for the entire axis of frequencies ($-\infty<\omega<+\infty$). Freedom from interference and accuracy of integrated systems ASD (Chapter 5) is investigated according to the relation precisely to this interference.

It is assumed that the receiver superheterodyne type radar with the amplitude detector.

Useful signal at the output UPCh is the radio pulses, fluctuating in the amplitude, which have duration $\tau_{\text{и}}$. The average/mean value of the amplitudes of envelope these impulses/momenta/pulses comprises $m_{\text{с}}=\langle u_{\text{с}}(t)\rangle$, where $u_{\text{с}}(t)$ - the amplitude pulse envelope on output of UPCh. Let us note that by duration of pulses $\tau_{\text{и}}$ is understood the duration of square pulses with the energy, which is contained in the real impulse/momentum/pulse. Subsequently we will use with these idealized square pulses. We assume that within the limits of the pulse duration their amplitude remains constant, in spite of the presence of the fluctuations of the echo signals. This assumption is justified by the fact that in the majority of the cases

the time of the correlation of the fluctuations of the amplitudes of the echo signals much more than $\tau_{и}$.

Considering detector inertia-free linear element/cell, let us consider two standard cases of the detection: linear and quadratic.

During the analysis of the passage of signal and interferences through the radio channel we will assume that the frequency characteristic of low-frequency amplifier (UNCh) is uniform in the region of low frequencies, i.e., it does not introduce the distortions of the form of video pulses, and it is equal to zero for the high frequencies: ω_0, $2\omega_0$ and so forth, where ω_0 - intermediate frequency. Therefore it is convenient to examine voltage not at the output of detector, but immediately at the output of UNCh.

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As is known, for example from [9], during the linear detection the average/mean value and the correlation function of output potential of UNCh during the action of radio pulse are respectively expressed by the formulas:

$$u_{1л}(t)=\frac{k_{1л}\sigma_п}{\sqrt{2\pi}}e^{-\frac{q_0^2(t)}{2}}\left\{I_0\left(\frac{q_0^2(t)}{2}\right)+\right.$$

$$\left.+q_0^2(t)\left[I_0\left(\frac{q_0^2(t)}{2}\right)+I_1\left(\frac{q_0^2(t)}{2}\right)\right]\right\}, \qquad (П.1)$$

$$B_{1л}(\tau)=\frac{k_{1л}^2\sigma_п^2}{8\pi}[b_1\rho_п(\tau)+b_2\rho_п^2(\tau)]. \qquad (П.2)$$

where $u_{1л}(t)$ - average/mean value of voltage/stress during the linear detection on the output of UNCh during the action of radio pulse;

$$q_0(t)=\frac{u_0(t)}{\sqrt{2}\sigma_п} \qquad (П.3)$$

- ratio of actual stress of signal to the effective value of interference at the output of detector (subsequently $q_0(t)$ will call simply relation signal/noise at the output of detector);

$$b_1=2\left\{q_0(t)e^{-\frac{q_0^2(t)}{2}}\left[I_0\left(\frac{q_0^2(t)}{2}\right)+I_1\left(\frac{q_0^2(t)}{2}\right)\right]\right\}; \qquad (П.4)$$

$$b_2=\left[e^{-\frac{q_0^2(t)}{2}}I_0\left(\frac{q_0^2(t)}{2}\right)\right]^2+\left[e^{-\frac{q_0^2(t)}{2}}I_1\left(\frac{q_0^2(t)}{2}\right)\right]^2; \qquad (П.5)$$

$I_m(x)$ - the Bessel function of the m order from the imaginary argument;

$k_{1л}$ - the constant coefficient, determining mutual conductance of linear detector;

$$\sigma_п^2=\sqrt{\frac{2}{\pi}}NB_0^2K\Delta f_{пр} \qquad (П.6)$$

- the dispersion of interferences at the output of UNCh;

$\rho_{\text{ш}} = e^{-5x^2\tau^2\Delta f_{\text{пр}}^2}$ - coefficient of correlation of the envelope of noise at the output UPCh;

B_0 - factor of amplification UPCh on the resonance frequent;

$\varkappa$ - constant coefficient $(\varkappa = 0.85)$;

$\Delta f_{\text{пр}}$ - bandwidth of UPCh at the level 0.707 on the voltage/stress.

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The feature above the letter indicates the operation of averaging on many realizations.

Formulas (P.1) and (P.2) are given for the case when $u_0(t)$ changes slowly in comparison with the time of the correlation of function $\rho_{\text{ш}}(\tau)$, i.e, the frequencies of the spectrum of fluctuations $u_0(t)$ are small in comparison with the bandwidth of the filter of intermediate frequency, which usually in actuality and occurs.

It must be noted that in expression (P.1) and further, where this is not stipulated specially, the averaging of output potential

of UNCh is produced only on radio interferences $(\sigma_{\text{п}})$, which are rapid fluctuations and by the smoothed servo system of range-only radar, then as on comparatively slow fluctuations of amplitudes of the echo signals of averaging, naturally, no.

With the weak signals, i.e., with $\bar{q}_0(t)<1$, the average/mean values of functions b_1 and b_2 are written/recorded as

$$\bar{b}_1 = \overline{2q_0^2(t)} - \overline{q_0^4(t)} + \frac{3}{8}\overline{q_0^6(t)} - \ldots.$$

$$\bar{b}_2 = 1 - \overline{q_0^2(t)} + \frac{11}{16}\overline{q_0^4(t)} - \frac{17}{48}\overline{q_0^6(t)} + \ldots.$$

With square-law detection the average/mean value of output potential of UNCh during the action of radio pulse and its correlation function is respectively equal to [9]:

$$u_{1\text{к}}(t) = k_2\sigma_{\text{п}}^2[1 + q_0^2(t)]; \qquad (\Pi.7)$$

$$R_{1\text{к}}(\tau) = k_2^2\sigma_{\text{п}}^4[\rho_{\text{п}}^2(\tau) + 2\overline{q_0^2(t)}\,\rho_{\text{п}}(\tau)]. \qquad (\Pi.8)$$

where $u_{1\text{к}}(t)$ - average/mean value of output potential of UNCh during the action of radio pulse with square-law detection; k_2 - the constant coefficient, which is determining mutual conductance of the square law detector.

In accordance with (P.2) and (P.8) for the dispersion of voltages/stresses $u_{1\text{л}}(t)$ and $u_{1\text{к}}(t)$, and also for the corresponding correlation coefficients we will obtain:

$$\sigma^2_{1\text{л}} = \frac{k^2_{1\text{л}}\sigma^2_{\text{п}}}{8\pi}(\text{б}_1 + \text{б}_2). \qquad (\Pi.9)$$

$$\rho_{1\text{л}}(\tau) = \frac{\text{б}_1}{\text{б}_1+\text{б}_2}\rho_{\text{п}}(\tau) + \frac{\text{б}_2}{\text{б}_1+\text{б}_2}\rho^2_{\text{п}}(\tau);$$

$$\sigma^2_{1\text{к}} = k^2_2\sigma^4_{\text{п}}[1 + 2\overline{q^2_0(t)}]. \qquad (\Pi.10)$$

$$\rho_{1\text{к}}(\tau) = \frac{2\overline{q^2_0(t)}}{1+2\overline{q^2_0(t)}}\rho_{\text{п}}(t) + \frac{1}{1+2\overline{q^2_0(t)}}\rho^2_{\text{п}}(t).$$

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The average/mean value of voltage/stress $u_{1\text{л}}(t)$ (or $u_{1\text{к}}(t)$, available at the output of UNCh during the action of radio pulse, it is the result of interaction of signal and interference, the pasts together through the nonlinear system (detector).

In this case by voltage of signal on the output of UNCh it is accepted to understand an increment in the average/mean value of resulting output potential UNCh, called by the appearance of a signal, since for the work of the temporary/time discriminator the vital importance has precisely an increment in the average/mean value of the resulting voltage/stress:

$$u_{\text{сл}}(t) = u_{1\text{л}}(t) - u_{\text{пл}}(t), \qquad (\Pi.11)$$

or

$$u_{\text{ск}}(t) = u_{1\text{к}}(t) - u_{\text{пк}}(t), \qquad (\Pi.12)$$

where $u_{\text{пл}}(t)$ and $u_{\text{пк}}(t)$ - average/mean values of output potential of UNCh with the signal, equal to zero, for linear and square-law detection

respectively.

Then the voltage of signal on the output of UNCh, in accordance with determination (P.11), is equal

$$u_{c\text{д}}(t) = K_{\text{пд}}(t)\,u_{\text{д}}(t). \qquad (\Pi.13)$$

where

$$u_{\text{д}}(t) = \frac{k_{1\text{п}}}{\sqrt{\pi}} u_0(t) - \qquad (\Pi.14)$$

- output potential of UNCh for the linear detection when at the input of the receiver of interference they are absent;

$$K_{\text{пд}}(t) = \frac{1}{2q_0(t)} e^{-\frac{q_0^2(t)}{2}} \left\{ I_0\left(\frac{q_0^2(t)}{2}\right) + q_0^2(t) \times \right.$$
$$\left. \times \left[I_0\left(\frac{q_0^2(t)}{2}\right) + I_1\left(\frac{q_0^2(t)}{2}\right)\right]\right\} \frac{1}{2q_0(t)}$$

- coefficient of suppression.

Coefficient $K_{\text{пд}}$ reflects/represents the suppression of noise signal in the linear detector.

with the weak signal, i.e., when $q_0(t)<1$,

$$K_{\text{пд}}(t) \approx \frac{1}{4} q_0(t) - \frac{1}{32} q_0^3(t) + \ldots$$

Approximation $K_{\text{пд}}$ only by first term with $q_0(t) \leqslant 1$ gives error not more than 10o/o [10]. If $q_0(t) \to \infty$, then $K_{\text{пд}} \to 1$.

With square-law detection without the signal, taking into account (P.7) we have

$$u_{\text{сж}}(t) = k_{2}\sigma_{\text{п}}^{2}. \qquad (\Pi.15)$$

Then the voltage of signal on the output of UNCh, in accordance with determination (P.12), is equal

$$u_{\text{сж}}(t) = \frac{k_2}{2} u_0^2(t). \qquad (\Pi.16)$$

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The voltage of signal on the output of UNCh with square-law detection does not depend on interference level and is proportional to the square of the amplitude of voltage on the input of detector. As a result of the fluctuations of the amplitudes of echo voltage signals on the input of detector $u_0(t)$ presents random process; therefore functions $K_{\text{вх}}(t)$, $u_{\text{сл}}(t)$, $u_{\text{сж}}(t)$ are also random.

For the determination of their statistical characteristics and, in particular, correlation functions, is required the knowledge of values $\bar{b}_1$ and $\bar{b}_2$ during the linear detection and $\overline{q_0^2(t)}$ with the quadratic. Values $\bar{b}_1$ and $\bar{b}_2$ in turn, are determined functions $\overline{q_0^2}(t)$, $\overline{q_0^4}(t)$ and so forth.

As examples let us consider two cases, frequently encountering in practice.

First case:

- $u_0(t)$ - normal stationary function with the correlation function

$$R_0(\tau) = \sigma_0^2 e^{-\alpha|\tau|} = \sigma_0^2 \rho_0(\tau). \qquad (\Pi.17)$$

moreover reception/procedure, that

$$m_0 = 3\sigma_0. \qquad (\Pi.18)$$

Here $R_0(\tau)$, $\sigma^2{}_0$, m_0 - respectively correlation function, dispersion and the average/mean value of voltage/stress $u_0(t)$.

It is not difficult to show that

$$\overline{q_0^2}(t) = \frac{m_0^2}{2\sigma_n^2} + \frac{\sigma_0^2}{2\sigma_n^2},$$

$$\overline{q_0^4}(t) = \frac{m_0^4}{4\sigma_n^4} + \frac{3m_0^2\sigma_0^2}{2\sigma_n^4} + \frac{3\sigma_0^4}{4\sigma_n^4}.$$

The second case:

- the fluctuations of the amplitudes of envelope the echo signals are subordinated to Rayleigh's law. Then $\sigma_0 \approx \frac{1}{2} m_0$, and

$$\overline{q_0^2}(t) = \frac{4}{\pi}[\bar{q}_0(t)]^2, \quad \overline{q_0^4}(t) = \frac{32}{\pi^2}[\overline{q_0(t)}]^4.$$

Thus, the determination of the statistical characteristics of output potential of UNCh with fluctuating signal at the input of detector in the general case presents the specific computational difficulties.

Measuring element/cell with the automatic following the temporary situation of the impulses/momenta/pulses, reflected from the target, is temporary/time discriminator (discriminator).

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Widespread is the temporary/time discriminator which consists of two amplifier stages, triggered to the period of the action of strobe pulses, two integrators and subtractor. In the temporary/time discriminator from the signal impulse/momentum/pulse by selecting devices/equipment are selected the sections, which coincide in the time with the selecting impulses/momenta/pulses, then is developed output voltage $u_\text{д}(t)$, proportional to a difference in the areas of the chosen sections of signal. Voltage/stress $u_\text{д}(t)$ is kept constant for a certain period of time, and then is dumped. With the arrival of the following pair of gates/strobes the operation is repeated above the next target pulse.

For the determination of the functional connection between the output voltage/stress of discriminator, the parameter of disagreement/mismatch $\Delta\tau = \tau_0(t) - \tau_c(t)$ and the interference level it is necessary to comprise and to solve system of equations, which describe the work of entire automatic control system upon consideration of the action of signals and radio interferences. (τ_c -

time position of the middle of strobe pulses). Since the discriminator is nonlinear element with the random parameters (due to the presence of the fluctuations of the amplitudes of the echo signals and radio interferences), obtaining and thereby the solution of this system of equations runs into the virtually insurmountable difficulties, especially upon consideration of interference effect with considerable intensity. This forces researchers to go to the way of using different approximation methods. In work [5] is proposed the method of replacing the real radio engineering devices/equipment, which are located under the effect of interferences, by the statistically equivalent filters (SEF), on input of which I operate the same transmitted communications/reports, that also in the real devices/equipment, and the output signals must coincide with the output signals of real devices/equipment in accordance with the selected criteria of approaching the random functions.

We will use this method during the analysis of temporary/time discriminator. For the practical targets the usually necessary and sufficient condition of the statistical equivalency of real discriminator and SEF is respectively the equality of their conditional mathematical expectations and conditional correlation functions of output voltages/stresses, i.e., mathematical expectations and the correlation functions, calculated when the parameter of disagreement/mismatch $\Delta\tau$ has the fixed value. In this

case are equal the corresponding unconditional moments/torques of the output voltages/stresses of real device/equipment and statistically equivalent filter.

Thus, the problem of the determination of the functional dependence of the output voltage/stress of discriminator $u_2(t)$ from $\Delta\tau$ and the radio interferences is reduced to finding of conditional mathematical expectations and conditional correlation functions of voltage/stress $u_2(t)$. Subsequently for the brevity in similar expressions word "conditional" we will omit.

We considered that the impulses/momenta/pulses at the input of discriminator have rectangular form and a duration τ_u. We will consider the selecting impulses/momenta/pulses also rectangular. When making these assumptions the dispersion of output potential of temporary/time discriminator is calculated, for example, in work [10].

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With the work in the linear section of characteristic of time discriminator, i.e., when $|\Delta\tau|<\frac{\tau_u}{2}$, is not difficult to show that the noise at the output of time discriminator does not depend on value $\Delta\tau_1$ and expression for the dispersion of noise during the quadratic

and linear detection are respectively equal to:

$$\sigma^2_{Дк} = \frac{k^2_Д c_1 \tau_и}{\Delta f_{пр}} \sigma^2_{1к2} + \frac{\sqrt{2} c_1 \tau_{сел} k^2_Д}{\Delta f_{пр}} \sigma^2_{1к1}, \qquad (П.19)$$

$$\sigma^2_{Дл} = \frac{\sqrt{2} c_1 \tau_{сел} k^2_Д}{\Delta f_{пр}} \sigma^2_{1л1} + \frac{c_1 \tau_и k^2_Д}{\Delta f_{пр}} \sigma^2_{1л2} + \\ + \frac{c_1 (2\tau_{сел} - \tau_и) k^2_Д}{\sqrt{2} \Delta f_{пр}} \sigma^2_{1л3}, \qquad (П.20)$$

where

$$\sigma^2_{1к1} = k^2_2 \sigma^4_п; \quad \sigma^2_{1к2} = 2k^2_2 \sigma^4_п \overline{q^2_0}(t);$$

$$\sigma^2_{1л1} = \frac{k^2_{1л} \sigma^2_п}{8\pi} \delta_2; \quad \sigma^2_{1л2} = \frac{k^2_{1л} \sigma^2_п}{8\pi} \delta_1;$$

$$\sigma^2_{1л3} = \frac{k^2_{1л} \sigma^2_п}{8\pi} (1 - \delta_2);$$

$k_Д$ - proportionality factor between the input and output voltage of discriminator; $\tau_{сел1}=\tau_{сел2}=\tau_{сел}$ - duration of the selecting pulses, they following directly one another; $\Delta f_{пр}$ - passband of the receiver of range-only radar; c_1 - constant coefficient (when $\varkappa=0{,}85$; $c_1=0.93$); $\tau_и$ - duration of the pulses, reflected from the target.

In accordance with the functional schematic of discriminator (discriminator) the mathematical expectation of the voltage of signal on its output following the arrival of the next pair of the selecting impulses/momenta/pulses is equal to:

$$u_{сД}(t) = k_Д \int_{t_0}^{t_0+\tau_{сел1}} u_c(t - \tau_0)\, dt - k_Д \int_{t_0+t_1}^{t_0+t_1+\tau_{сел.2}} u_c(t - \tau_0)\, dt. \qquad (П.21)$$

where $u_c(t-\tau_0)$ - voltage of signal on the input of discriminator, determined by expression (P.11) or (P.12).

If the amplitude of pulses $u_c(t)$ in the limits of the durations of gates/strobes is considered constant/invariable, then sufficiently simply can be obtained expressions for $u_{cx}(t)$ with different relationships/ratios of the durations of the echo pulses and gates/strobes.

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Thus, when $\tau_{cex1}=\tau_{cex2}=t_1=\tau_{cex}=\tau_x$ voltage/stress $u_{cx}(t)$ is equal to:

$$u_{cx}(t)=k_Д u_c(t)\times\begin{cases} 2\Delta\tau \ \text{при} \ 0\leqslant|\Delta\tau|\leqslant\frac{\tau_x}{2}, \\ \frac{3}{2}\tau_x-\Delta\tau \ \text{при} \ \frac{\tau_x}{2}\leqslant|\Delta\tau|\leqslant\frac{3\tau_x}{2}, \\ 0 \ \text{при} \ |\Delta\tau|\geqslant\frac{3\tau_x}{2}. \end{cases} \quad (П.22)$$

Key: (1). with.

In the presence of interferences expression (P.22) remains valid only for the linear sections of the characteristics of discriminators, since in other sections $|\Delta\tau|>\frac{\tau_x}{2}$, and noises at the output of devices/equipment in this case are not additive.

On the basis of expression (P.22) the mathematical expectation of the voltage of signal at the output of the time discriminator

during the action of impulse/momentum/pulse for the linear section of its characteristic can be recorded in the form

$$u_{\text{д}}(t) = 2k_{\text{д}}u_{\text{c}}(t)\,\Delta\tau.$$

After using dependences (P.13), (P.14) and (P.16), we will obtain

$$u_{\text{дл}}(t) = \frac{2k_{1л}k_{\text{д}}}{\sqrt{\pi}}K_{\text{пл}}(t)\,u_{\text{ф}}(t)\,\Delta\tau \qquad (\Pi.23)$$

- during the linear detection,

$$u_{\text{дк}}(t) = k_{\text{к}}k_{\text{д}}u_0^2(t)\,\Delta\tau \qquad (\Pi.24)$$

- with quadratic detection.

From formulas (P.23) and (P.24) it follows that $u_{\text{д}}(t)$ *is a result* of the linear transformation of the parameter of disagreement/mismatch $\Delta\tau$ with the random transmission factor, determined by the fluctuations of voltage/stress $u_{\text{ф}}(t)$.

Since in the present work integrated systems are investigated by the methods of the theory of the continuous servo systems, then the obtained above statistical characteristics of pulse output potentials of time discriminator should be replaced the appropriate statistical characteristics of the voltages/stresses of equivalent continuous device/equipment.

As it was shown, interferences at the output of discriminator are the sequence of the impulses/momenta/pulses, which have one and

the same form, but random amplitude, with the zero mathematical expectation and the dispersion, by expressed formulas (P.19) and (P.20). Duration of disturbing pulses is determined by the time of the arrival of signal impulse/momentum/pulse and by the moment/torque of jettisoning output potential of discriminator. Let it be equal to τ_1.

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Since is examined the mode/conditions of target tracking, the from one period to the next displacement/movement of disturbing pulses at the output of time discriminator is possible only in the limits of the durations of the selecting pulses. Taking into account that $\tau_{\text{сел}}$ T, where T - repetition period of the selecting impulses/momenta/pulses, it is possible to consider disturbing pulses as those having repetition period T.

The function of the sequence of the equidistant impulses/momenta/pulses, which have identical duration and random amplitude, as is known, for example, from [6] is different

$$R_f(\tau)=\begin{cases}\sigma_f^2\left(1-\frac{|\tau|}{\tau_1}\right), & \overset{(1)}{\text{при}}\ |\tau|\leqslant\tau_1,\\ 0, & \overset{(1)}{\text{при}}\ |\tau|>\tau_1.\end{cases}\qquad (\Pi.25)$$

Key: (1). with.

where

$$\sigma_f^2 = \sigma_\text{д}^2 \frac{\tau_1}{T} \qquad (\Pi.26)$$

- the dispersion of the pulse sequence; $\sigma_\text{д}^2$ - dispersion $\sigma_{\text{дк}}^2$ or $\sigma_{\text{кд}}^2$, determined by expressions (P.19) and (P.20) for the quadratic and linear detection respectively.

Let us note that formula (P.25) corresponds to the continuous part of the spectrum of the sequence of impulses/momenta/pulses; the discrete/digital part of it spectrum is not considered, since all periodic components lie/rest out of the passband of servo system.

However, the signal pulses at the output of time discriminator have the same duration τ_1 and the amplitude, determined by dependences (P.23) and (P.24). The average/mean value of the voltage of the signal of equivalent continuous system can be recorded as

$$u_{\text{дэ}}(t) = u_\text{д}'(t) \frac{\tau_1}{T}. \qquad (\Pi.27)$$

As it was shown, time discriminator possesses linear transformative properties with respect to the transmitted parameter $\Delta\tau(t)$; therefore equation of the statistically equivalent filter, which replaces real discriminator, will take the form:

$$u_\text{д}(t) = \lambda(t)\,\Delta\tau(t) + f_\text{р}(t),$$

where $f_\text{р}(t)$ - normal stationary random function, which characterizes additive noise at the output of discriminator; $\lambda(t)$ - the random function, which characterizes transmission properties of time

discriminator of the transmitted parameter $\Delta\tau(t)$.

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Function $f_p(t)$ has zero mathematical expectation and correlation function, determined by expression (P.25). On the basis of expressions (P.23), (P.24) and (P.27) for the random function $\lambda(t)$ in the case of quadratic and linear detection it is possible to record respectively:

$$\lambda_{\kappa}(t) = k_{\partial}k_{\Pi}u_{c\Pi}^{2}(t)\frac{\tau_1}{T}, \qquad (\Pi.28)$$

$$\lambda_{\pi}(t) = \frac{2k_{1\pi}k_{\pi}}{\sqrt{\pi}}K_{\Pi\pi}(t)u_{c}(t)\frac{\tau_1}{T}. \qquad (\Pi.29)$$

If the measured parameter is characterized by the measuring error of range $\Delta Д(t) = Д(t) + Д_{a}(t) - Д_{\kappa}(t)$, then equation of time discriminator for the linear section of its characteristic takes the form

$$u_{\partial}(t) = K_{\partial p}(t)\Delta Д(t) + f_{p}(t). \qquad (\Pi.30)$$

where

$$K_{\partial p}(t) = \frac{\lambda(t)}{k_{1\pi}}, \qquad (\Pi.31)$$

$$k_{1\pi} = 150\frac{^{(1)}м}{мксек}.$$

Key: (1). m/μs.

Expression (P.31) value $\lambda(t)$ is equal either $\lambda_{\kappa}(t)$. or $\lambda_{\pi}(t)$ for the case of quadratic or linear detection respectively.

Thus, time discriminator with the work in the linear section of its characteristic is equivalent to inertia-free linear component/link with the random transmission factor $\overset{K_{\partial p}(t)}{\wedge}$ and to source

of additive noise $f_p(t)$. Let us find density of distribution and moments/torques of random function $K_{sp}(t)$.

The transmission factor of temporary/time discriminator with square-law detection taking into account (P.28) and (P.31) is written/recorded as

$$K_{\text{врк}}(t) = K_{\text{кв}} u_0^2(t). \qquad (\Pi.32)$$

where

$$k_{\text{кв}} = \frac{k_2 k_{\text{л}}}{k_{1\text{в}}} \cdot \frac{\tau_1}{T}, \qquad (\Pi.33)$$

and during the linear detection taking into account (P.29) and (P.31) has the form

$$K_{\text{врл}} = k_{\text{л}} K_{\text{дл}}(t) u_0(t). \qquad (\Pi.34)$$

where

$$k_{\text{л}} = \frac{2k_{1\text{д}} k_{\text{л}}}{\sqrt{2}\, k_{1\text{в}}} \cdot \frac{\tau_1}{T} \qquad (\Pi.35)$$

In a small relation signal/noise ($\overline{q}_0(t) \lesssim 1$)

$$K_{\text{дл}} = \frac{1}{4} q_0(t).$$

then

$$K_{\text{врл}}(t) = \frac{k_{\text{л}}}{4\sqrt{2}\sigma_{\text{ш}}} u_0^2(t);$$

in the large relation signal/noise $K_{\text{дл}}(t) \approx 1$ and

$$K_{\text{врл}}(t) = k_{\text{л}} u_0(t).$$

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During the analysis of the statistical characteristics of the

coefficient of transmission of the time discriminator is expedient to consider two cases:

- the case of linear detection with the large signal, i.e., when

$$K_{\text{вр}}(t) = K_{\text{вр1}}(t) = k_{\text{д}} u_{0}(t); \qquad (\text{П}.36)$$

- case of linear detection with the weak signal, and also square-law detection, i.e., when

$$K_{\text{вр}}(t) = K_{\text{вр2}}(t) = k_{\text{кв}} u_0^2(t). \qquad (\text{П}.37)$$

Expressions (P.36) and (P.37) show that the transmission factor of time discriminator as a result of the fluctuations of the amplitudes of the echo signals is the random function the law of distribution of which with square-law detection does not depend on the level of radio interferences, whereas during the linear detection it is the nonlinear function of the latter.

For computing the measuring errors of range by integrated ranging systems in the general case is required the knowledge of the laws of the distribution of random function $K_{\text{вр}}(t)$.

With a change in the transmission factor of temporary/time discriminator in the first case its law of distribution remains the same as in voltage/stress $u_0(t)$.

As an example above we examined the case, when $u_0(t)$ is a normal

stationary random function with the correlation function and the average/mean value, determined expressions (P.17) and (P.18). Then for the statistical characteristics of transmission factor of time discriminator can be recorded:

$$m_1 = \langle K_{\text{вр}1}(t) \rangle = k_д m_0 = 3k_д \sigma_0. \qquad (\Pi.38)$$

$$R_1(\tau) = \langle [K_{\text{вр}1}(t) - m_0][K_{\text{вр}1}(t+\tau) - m_0] \rangle = \sigma_1^2 \rho_1(\tau), \qquad (\Pi.39)$$

where

$$\sigma_1^2 = k_д^2 \sigma_2^0; \qquad \rho_1(\tau) = \rho_0(\tau) = e^{-\alpha|\tau|}.$$

It is not difficult to see that $m_1 = 3\sigma_1$.

If $u_0(t)$ is subordinated to Rayleigh's law, then taking into account the fact that $\sigma_0 \sim m_0/2$, for average/mean value $K_{\text{вр}1}(t)$ and its root-mean-square value is performed equality $\sigma_1 \sim m_1/2$.

With a change in the transmission factor of time discriminator in the second case its distribution function is the function of the distribution of the square of normal random function. If $u_0(t)$ - normal stationary function, then the one-dimensional function of the distribution of its square takes the form (for example, see [6]):

$$W(u_0^2) = \frac{1}{\sqrt{2\pi u_0^2}\, 2\sigma_0}\left[e^{-\frac{\left(\sqrt{u_0^2} - m_0\right)^2}{2\sigma_0^2}} + e^{-\frac{\left(\sqrt{u_0^2} + m_0\right)^2}{2\sigma_0^2}}\right].$$

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Since $K_{\text{вз}}(t) = k_{\text{кв}} u_0^2(t)$, then $W(K_{\text{вз}}) = \frac{1}{k_{\text{кв}}} W(u_0^2)$, where $\frac{1}{k_{\text{кв}}}$ - jacobian of transformation.

It is possible to show that in this case

$$m_2 = \langle K_{\text{вр2}}(t) \rangle = k_{\text{кв}} (m_0^2 + \sigma_0^2),$$

$$R_2(\tau) = \langle [k_{\text{кв}} u_0^2(t) - m_2] [k_{\text{кв}} u_0^2(t+\tau) - m_2] \rangle = \sigma_2^2 \rho_2(\tau),$$

where

$$\sigma_2^2 = 4k_{\text{кв}}^2 m_0^2 \sigma_0^2 + 2k_{\text{кв}}^2 \sigma_0^4; \quad \rho_2(\tau) = \rho_0(\tau) \frac{\sigma_0^2 \rho_0(\tau) - 2m_0^2}{\sigma_0^2 + 2m_0^2}.$$

In the case of $m_0 = 3\sigma_0{}^2$ in question, then

$$m_2 \approx 1{,}65\sigma_2, \quad \rho_2(\tau) \approx 0{,}95\rho_0(\tau) + 0{,}05\rho_0^2(\tau) = \rho_0(\tau).$$

Latter/last expression shows that in the case of $m_0 = 3\sigma_0$ the coefficient of correlation of function $K_{\text{вр2}}(t)$ with an accuracy to several percentages is equal to the coefficient of correlation of voltage/stress $u_0(t)$.

If voltage/stress $u_0(t)$ has a law of Rayleigh distribution, then the function of distribution $K_{\text{вр2}}(t)$, which is subordinated to the distribution of the square of Rayleigh random function, is the exponential distribution for which are valid the relationships/ratios (for example, see [6]):

$$m_2 = \sigma_2 = \frac{4}{\pi} m_0^2 \quad \text{and} \quad \rho_2(\tau) = \rho_0^2(\tau).$$

Thus, if during the work of the integrated ranging system modulus of the parameter of disagreement/mismatch $|\Delta\tau| \leqslant \frac{\tau_{и}}{2}$ when $(\tau_{и} = \tau_{стр})$. which is characteristic for the mode/conditions of tracking, then time discriminator during the analysis can be replaced to the linear inertia-free amplifier with random coefficient $K_{вр}(t)$ and the source of additive noise $\xi_{р}(t)$. The chance of transmission factor of time discriminator $K_{вр}(t)$ depends on the fluctuations of the amplitudes of the echo signals the law of distribution of which with square-law detection is determined by the law of the distribution of the square of envelope the echo signals and does not depend on the level of radio interferences, whereas during the linear detection it is the nonlinear function of jamming intensity.

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